Hector Zenil
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Irreducibility and Computational Equivalence

10 Years After Wolfram's A New Kind of Science

Springer

Hector Zenil (Ed.)

Irreducibility and Computational Equivalence

It is clear that computation is playing an increasingly fundamental role in the development of mathematics, as well as in the natural and social sciences. The work of Stephen Wolfram over the past decades has been a salient part of this phenomenon, laying the foundations for the field of Complex Systems. Many of the constructs and ideas developed in his book A New Kind of Science (ANKS) have become part of scientific discourse and general academic knowledge – from the now established Elementary Cellular Automata to the unconventional concept of mining the Computational Universe, from his widely used Behavioural Classification to his principles of Irreducibility and Computational Equivalence.

This volume, with a Foreword by Gregory Chaitin and an Afterword by Cris Calude, covers these and other topics related to or motivated by Wolfram's seminal ideas, reporting on research undertaken in the decade following the publication of his ANKS book. Featuring 39 authors, its 23 contributions are organized into seven parts:

- Mechanisms in Programs & Nature
- Systems Based on Numbers & Simple Programs
- Social and Biological Systems & Technology
- Fundamental Physics
- The Behavior of Systems & the Notion of Computation
- Irreducibility & Computational Equivalence
- Reflections & Philosophical Implications.

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Chapter 17
Wolfram's Classification and Computation in Cellular Automata Classes III and IV

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Abstract. We conduct a brief survey on Wolfram's classification, in particular related to the computing capabilities of Cellular Automata (CA) in Wolfram's classes III and IV. We formulate and shed light on the question of whether Class III systems are capable of Turing-completeness or may turn out to be "too hot" in practice to be controlled and programmed. We show that systems in Class III are indeed capable of computation and that there is no reason to believe that they are unable, in principle, to reach Turing universality.

Keywords: cellular automata, universality, unconventional computing, complexity, gliders, attractors, Mean field theory, information theory, compressibility.

1 Wolfram's Classification of Cellular Automata

A comment in Wolfram's *A New Kind of Science* gestures toward the first difficult problem we will tackle (ANKOS) (page 235): *trying to predict detailed properties of a particular cellular automaton, it was often enough just to know what class the cellular automaton was in.* The second problem we will take on concerns the possible relation between complexity of Cellular Automata and Turing universal computation, also highlighted by Wolfram in his ANKOS (page 691 – on Class 4 behaviour and Universality): *I strongly suspect that it is true in general that any cellular automaton which shows overall class 4 behaviour will turn out—like Rule 110—to be universal.* The classification and identification of cellular automata (CA) has become a central focus of research in the field. In

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