

# An Approach to Knowledge and Belief Based on Kleene Strong Logic

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## Tesis presentada en opción al grado de Doctor en Ciencias de la Computación por:

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## 1. Overview

The aim in this thesis is the formal treatment of representation and reasoning of knowledge and belief. Main problem to overcome this challenge are the lack of formalisms being flexible enough in order to deal with the inherent dynamic of any epistemic process. Both, knowledge and belief, involve to handle well defined information as well as ambiguous, incomplete or undefined.

In this thesis, an epistemic logic intended to go on the solution of mentioned problem is developed. The representation framework, called *epistemic context* is a specific structure of Kripke [Kripke, 59], namely, a set of partially ordered possible worlds. The logic underling it is the Kleene Strong Three-Valued Logic [Kleene, 52], having *true*, *false* and *undefined* truthvalues. The paramount in our proposal is that knowledge and belief are gradually obtained, having a context ever.

The first major epistemic approach, being ruling so far, is due to J. Hintikka [Hintikka, 62], which is supported on the Modal Logic and the Semantic of Possible Worlds. Following Hintikka, a world is a *description of the state of the things*. Following Kripke, the Semantic of Possible Worlds provides to modal logic the possibility to talk not only about the present world, but on any possible world. Briefly, for Hintikka, knowledge is each *true* statement in every world, while belief is each *true* statement in at least a world.

The drawbacks of the Kripke-Hintikka paradigm, are the so called *logical omniscience* and *ideal reasoning*, which

assume agents having unlimited resources and exhaustive not fails capabilities of reasoning. Thus, computational agents or human beings having material and temporal limitations, cannot be well modelled using that formalism.

Essentially, logic omniscience is the acceptance that every theorem is knowledge, which is due to the necessitation rule. As for ideal reasoning is concerned, it engages the inference of every theorem without exception, and is due to the modal distributivity axiom. However, this distinction is not present in the literature so far, but erroneously both are identified as the same.

A general characterisation of logic omniscience and ideal reasoning are given in this thesis. Then, the advantages and limitations of major works having the purpose to overcome logic omniscience and ideal reasoning are analysed. For instance, in [Thijsse, 92], *types* of logic omniscience are suggested. However, based on present contents of this thesis, it is straightway shown that these are either axioms or inference rules conveying ideal reasoning, in absolute or relative manner, actually. Furthermore, it is shown that logic omniscience is a particular case of ideal reasoning.

The extended use of modal logic and the Semantics of Possible Worlds (SPW) dealing with knowledge and belief formalisation is due to the suggestive image they provide, that knowledge and believes statements in current world depend on truthness in the conceivable (possible, accessible) worlds from the current.

The point is that, as far as we understand, the related current approaches do not capture, in an adequate formal way, the intuitive idea of a possible or accessible world. In the kripkean semantics of worlds, a rather arbitrary relation of possibility determines the possible worlds for the current. As we will show later, sometimes, that fact conveys that *true* statements in the current world cannot be knowledge in this world, which is a counter-sense.

The reason of mentioned counter-sense roots in the arbitrariness of the relation of possibility between worlds. In our epistemic contexts, a world is possible if it contains at least the same information that the current world. The relation of possibility is a partial order of *information*. In this manner, *true* (*false*) statements in the current world are maintained in every possible world, and they are knowledge in the current world.

On the other hand, given an epistemic context of worlds, the more informed the possible worlds are, the more knowledge and belief is generated in the current world in the context.

Modelling knowledge and belief acquisition require to operate both with defined information as well as with incomplete or badly defined information, as mentioned before. Thus, in epistemic logic, it seems better to set formalisms with a kernel of precise inference rules and axioms being applied on well defined information. Though on the other hand, these formalisms should provide enough flexibility to be applied over not completely determined information. Moreover, it is desirable that they can be extended in order to deal with issues not considered still.

Kleene Strong Three-Valued Logic arises as a flexible formalism to back the dynamic character of knowledge and belief acquisition. In the *Epistemic Contexts*, well defined information is *true* and *false*, while the rough information is *undefined*. The knowledge in a world in this context is the *true* statements in every and most informed world, while believes are the *true* statements in at least one world. Thus, knowledge is belief, but not conversely.

An important remark is that the mentioned definition of knowledge is quite different and more general than the usual definitions in the literature, for which knowledge is the *true* statement in *every* world. With the definition proposed here, modelling evolving information is done in such a way that information not considered knowledge in the initial world, can become knowledge if there is *true* information supporting it. This must occur in the most informed worlds. This corresponds to a dynamic process of knowledge acquisition. Formalisation is through modal operators of knowledge **K** and belief **B**. So far we have referred to the semantic aspect of the epistemic logic.

Syntactical characterisation of our epistemic logic is based

on Gentzen Sequent Calculus. Sequents allow to deal with inference between sets of formulae rather than with formulae alone. This fact conveys the advantage to treat with formulae (information) having different logical status, which is very useful for handling well defined information as well as undefined or incomplete.

Formal account of mechanisation is based on the Analytic Tableaux demonstration method. The *constructive-like* manner to perform derivations and proofs of theorems in tableau, is adequate for automating the knowledge and belief acquisition in an Epistemic Context. In addition, either declarative or operative information can be added at every step of the inference process. Finally, there is a direct translation from sequents to tableaux and resolution method over tableaux has been defined in [Ohlbach, 88].

The dynamics of knowledge and belief acquisition strongly concerns its own *change* as soon as new information is added. The ruling formal proposals of the so called *belief change*, are *Revision Theory* [Alchourron et al. 86] and *Belief Updating* [Katsuno and Mendelzon, 91].

The problem within this area, which will be studied in this thesis, is the non-intuitive changes resulting from the presence of incomplete or ambiguous information.

Change of knowledge conveys change of worlds because in epistemic contexts, knowledge are the true statements in most informed worlds. The change of a world is caused by modifying its *true* or *false* statements which are in conflict with *true* or *false* information added to this world. In our approach, by its own nature, *undefined* statements are not in conflict with any information.

The first step of statements change in a world, is by eliminating these statements from the worlds (semantically, changing *true* or *false* truth-values to *undefined*). This corresponds to the notion of *dis-learning* of non actual information, indicated by the presence of new information. Thereafter, regarding the intended change, elimination is confirmed, or inclusion of the negated statements is allowed (semantically, statements remain *undefined* or become *false*).

Notice that change of information in an epistemic context begins in a world but can imply a change in the precedent or further worlds in the contexts as well. In fact, every world containing statements in conflict with new information should be revised or updated, which in turns conveys revision or updating of all the epistemic context.

In the epistemic context, the new worlds (revised or updated) are generated from a conflictless world with new information. The localisation of the different sequences of worlds that should be changed is not difficult using the structure of the context. Therefore, context's structure makes easy the localisation of these worlds.

The general specification of an algorithm devoted to

localise the worlds plausible to change will be outlined at the ending of this thesis. The generality of the algorithm specifications allows the introduction of any operator of change as a parameter of the algorithm.

## 2. Concluding Remarks

Main remarks from this thesis is the relativity of the following aspects concerning knowledge and belief.

- Knowledge and beliefs are determined by the available information, which constitutes a context. In other words, knowledge and beliefs are relative to a given context of information.
- The information can be threekinds: true, false or ambiguous (incomplete, undefined, noisy).
- As long as *undefined* information becomes *true* or *false*, knowledge or beliefs can increase, and the converse, as long as *true* or *false* information evolves in ambiguous (*undefined*) one, knowledge or beliefs can decrease.
- From the aforementioned, any formalism dealing with knowledge and belief, should account the treatment of both kind of information.
- Since a formal point of view, Ideal Reasoning is relative to the rules of inference in the formal system.
- Hence, the Logic Omniscience, which is implied by Ideal Reasoning, it also depends on the subjacent formal system.

## 3. Major Thesis Contributions

- A general characterisation of Logic Omniscience.
- A general characterisation of Ideal Reasoning
- The consideration of Logic Omniscience as a particular instance of Ideal Reasoning.
- A Constructive-Like Semantic of Possible Worlds based on Kleene Strong Three-Valued Logic.
- Definition of an Information Logic Consequence generalising the classical.
- Definition of Epistemic Context.
- Definition of Knowledge and Belief over an Epistemic Context.
- Modelling of Knowledge and Belief growth in a constructive-like manner.
- Definition of an Information Epistemic Logic being complete and sound.
- An algorithm for the change (revision and updating) of knowledge and belief over Epistemic Contexts.

- Optimisation for the change of Knowledge and Belief.

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