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The Dirac equation in *D*-dimensional spherically symmetric spacetimes

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Abstract
We expound in detail a method frequently used to reduce the Dirac equation in *D*-dimensional (*D≥4*) spherically symmetric spacetimes to a pair of coupled partial differential equations in two variables. As a simple application of these results we exactly calculate the quasinormal frequencies of the uncharged Dirac field propagating in the *D*-dimensional Nariai spacetime.

Keywords: Dirac field; Nariai spacetime, Spherically symmetric, Quasinormal modes.

Resumen
Exponemos con detalle un método frecuentemente usado para simplificar la ecuación de Dirac en espaciotiempos esféricamente simétricos en *D*-dimensiones (*D≥4*) a un par de ecuaciones diferenciales parciales en dos variables. Como una aplicación directa de estos resultados calculamos las frecuencias cuasinormales del campo de Dirac sin carga en el espaciotiempo *D*-dimensional de Nariai.

Palabras clave: Campo de Dirac, espaciotiempo de Nariai, esféricamente simétrico, modos cuasinormales.

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I. INTRODUCTION

Recently in many research lines of theoretical physics the models in which the spacetime has more dimensions than the four dimensions observable in our daily experience have been studied extensively. The most analyzed models are those related to string theory [1]. Also the scrutiny of the properties and solutions of higher dimensional general relativity has attracted a lot of attention (see Ref. [2] and references therein). In several of these research lines we need to know the classical properties of the higher dimensional spacetimes to examine different phenomena. Therefore the investigation of these classical properties is an active research field.

To analyze the classical properties of a given spacetime a common method is to use a field as probe [3, 4]. Thus in the past several scattering phenomena of classical fields were studied, in order to know how to calculate the physical parameters of the spacetime from the measured values of the physical quantities corresponding to the classical field.

The quasinormal modes (QNMs) are solutions to the equations of motion for a classical field that satisfy the radiation boundary conditions that are natural in the spacetime in which the field is propagating [3, 4]. For example, in asymptotically flat black holes the boundary conditions of the QNMs are that the field is purely ingoing near the event horizon and purely outgoing near infinity [3]. For asymptotically anti-de Sitter black holes we impose the boundary condition that the field vanishes at infinity and is ingoing near the event horizon.

It has been shown that the QNMs are a useful tool to calculate the physical parameters of a spacetime [3, 4]. Hence if we know the quasinormal frequencies (QNF) of a classical field we can infer the values of several physical quantities of the spacetime such as its mass, charge, and angular momentum [3]. Furthermore it has been proposed that the QNMs encode some information about the quantum properties of the black holes [5].

To compute the QNF of a classical field in a given spacetime the usual procedure is to reduce the equations of motion for the field to a radial ordinary differential equation (assuming a given dependence on the angular variables and a harmonic time dependence) and impose to the radial function the boundary conditions of the QNMs.

Also notice that the reduced form of the equations of motion is useful (and sometimes necessary) to study many other classical or semiclassical phenomena. Thus we believe that at present time the understanding of the separability properties of the equations of motion for classical fields in higher dimensional curved spacetimes must be a relevant part in the education of a physicist.

Motivated by these theories that assume a number of spacetime dimensions greater than four, the separability properties of the equations of motion for several classical