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On the fragility of an internet-based dialogue*

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Abstract
This paper examines the computer mediated asynchronous interaction of a group of in-service mathematics teachers who are exchanging their points of view on the solution of a mathematical activity. These teachers are enrolled in a master's degree program in mathematics education. Based on the analysis of the interaction, it is argued that this group of teachers accomplished a dialogue (as defined in Alrø & Skovsmose, 2002), which helps to produce a positive change in some of the mathematical ideas of one of the participants in the dialogue. The analysis illustrates how the involvement of the teacher educator in the interaction may have an influence capable of breaking the dialogue established between the teachers.

Keywords
Mathematics teacher education, eLearning, interaction, communication, dialogue, critical learning.

Sur la fragilité d'un dialogue basé sur l'Internet[a]

Résumé
Cette étude examine l'interaction asynchrone médiatisée par ordinateur d'un groupe de mathématiques en service des enseignants qui échangent leurs points de vue sur la solution d'une activité mathématique. Ces enseignants sont inscrits dans un programme de maîtrise en enseignement des mathématiques. Sur la base de l'analyse de l'interaction, il est affirmé que le groupe d'enseignants a accompli un dialogue (tels que définis dans Alrø & Skovsmose, 2002), ce qui contribue à produire un changement positif dans certains des idées mathématiques de l'un des participants à la dialogue. L'analyse montre comment l'implication de l'enseignant, éducateur dans l'interaction mais ont une influence capable de rompre le dialogue établi entre les enseignants.

Mots-clefs
Mathématiques de formation des enseignants, l'apprentissage, l'interaction, la communication, le dialogue, la critique d'apprentissage.

Sobre la fragilidad de un diálogo basado en internet[b]

Resumen
En este artículo se examina la interacción asincrónica mediada por computadora entre un grupo de docentes de matemáticas en servicio —inscrito en la maestría de matemática educativa— que intercambia puntos de vista sobre la solución de una actividad. Con base en el análisis de la interacción, se argumenta que el grupo establece un diálogo —en el sentido de Alrø & Skovsmose, 2002—, que contribuye al cambio positivo en algunas de las ideas matemáticas de uno de los participantes. El análisis ilustra que la participación del formador de profesores en la interacción puede ejercer una influencia capaz de romper el diálogo establecido entre los docentes.

Palabras clave
Formación de profesores de matemáticas, educación a distancia, interacción, comunicación, diálogo, aprendizaje crítico.

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Introduction

This report is part of the PhD research project entitled *How to stimulate rich interactions and reflections in internet-based mathematics teacher education?* that is currently being developed at the University of Roskilde in Denmark. All the empirical data used in this project, have been taken from an nternet-based mathematics education program called Prome-Cicata. This is a program that offers Master's and PhD studies to in-service mathematics teachers from different Latin American countries, and working in different educational levels —basic, lower secondary, upper secondary and university level. This educational program is sponsored by the Instituto Politécnico Nacional of México, one of the largest public universities in Mexico.[1]

In general, the Prome-Cicata program aims at introducing in-service mathematics teachers to the academic field of mathematics education. To introduce the teachers to the mathematics education theories, its research methods and results, is a way of providing them with a set of "lenses". A basic assumption of the program is that those lenses will allow them to revisit and to have a new view of their own *school mathematics culture*, the one constituted by their understanding and beliefs about the content that they teach, their students, their role as educators in society, about their institution.

The way of introducing teachers to the mathematics education field is closely linked to a communication process; that is to say, Prome-Cicata's way of delivering the *math education lenses* to the teachers is through readings, mathematical activities, video and audio files, that should be analyzed and/or solved, but also discussed, criticized and reflected upon it. This introduction process —or enculturation process— is not a straightforward one. It is common to find resistance, skepticism and doubts among teachers. It is necessary then to open channels of communication and interaction that will allow us to express, to share, to compare, to criticize and to be aware of our ideas and feelings. As Cooney (1994, p. 109), affirms: *[O]ur beliefs about teaching are shaped by social situations and therefore can only be reshaped by social situations.* Hence, communication and interaction become key elements of this process.

Thus, although in general it can be argued that my project wants to increase our knowledge about how to foster "rich" interactions in the educational setting previously described, it is necessary to clarify the aim of this paper in more precise terms. In the next section I shall talk about the theoretical framework that I used to structure my research project, as well as this writing.

Theoretical framework

The empirical data for this research project are mainly constituted by the registers of asynchronous interactions teachers and teacher educators. An asynchronous interaction is the one that is carried out mainly by means of an exchange of written messages between two or more people, but where the feedback and reactions to the messages are not immediate. The asynchronous interactions usually last several days, allowing participants to have more time to formulate their opinions and to reflect on comments and opinions expressed by the other participants. It is even possible to consult external sources in order to enrich and clarify a discussion in an asynchronous communication. The email messages and the discussion forums are some examples of asynchronous communication.

Those asynchronous interactions have been analyzed using the Inquiry Co-operation model (IC-Model), of Alrø & Skovsmose (2002). The model, strongly rooted in the critical mathematics education approach (Skovsmose, 1994), argues that in order to have a fruitful interaction, it should be based on mutual respect, on the willingness to make public our ideas and subject them to scrutiny, as well as in a real interest to listen and analyze our interlocutor's ideas.

The IC-Model is constituted by a set of communicative characteristics. According to this theoretical approach, an interaction as the previously described should have several of these communicative characteristics. In fact when those characteristics are present in an interaction, it is regarded as a special kind of interaction called *dialogue* that possesses the potential to serve as a basis for critical learning and reflection. Because I am working with non-novice teachers, who have a certain vision about their *school mathematics culture*, it is desirable to establish dialogues with them and among them, to explain and to identify different ideas and beliefs about their mathematical culture, to reflect upon them, and to make a critical reading of them.

The communicative characteristics that define a dialogue are *getting in contact*, *locating*, *identifying*, *advocating*, *thinking aloud*, *reformulating*, *challenging* and *evaluating*; and they could be succinctly defined as follows: *[G]etting in contact involves inquiring questions, paying attention, tag questions, mutual confirmation, support and humour. Locating has been specified with the clues of inquiring, wondering, widening and clarifying questions, zooming in, check-questions, examining possibilities and hypothetical questions. Identifying involves questions of explanation and justification and crystallizing mathematical ideas. Advocating is crucial to the particular trying out of possible justifications, and it is closely related to arguing and considering. Thinking aloud often occurs as hypothetical questions and expression of thoughts and feelings. Reformulating can occur as paraphrasing, completing of utterances and staying in contact. Challenging can be made through hypothetical questions, examining new possibilities, clarifying perspectives, and it can be a turning point of investigation. Evaluating implies constructive feedback, support and critique* (Alrø & Skovsmose, 2002, p. 110).

1 More information about this in-service mathematics education program can be found in: http://www.matedu.cicata.ipn.mx

It is important to clarify that the theoretical concept of *dialogue* is not used as a synonym for *rich interaction*. The richness of an interaction can be measured in terms of the kind of reflections that it produces. If the reflections produced during an interaction helps the teacher —or even the teacher educator— to understand, identify, explain or criticize any element of its own *school mathematics culture*, then it can be regarded as rich. Other analysis of asynchronous interactions between in-service mathematics teachers that I have made prior to the preparation of this writing (Sanchez, 2008), suggests that an interaction that is modulated by a dialogue may work as a basis for the emergence of such reflections. The data that I will show in this writing are an addition to the empirical evidence that supports this hypothesis.

This writing presents an analysis of an asynchronous interaction using the so-called IC-Model (Alrø & Skovsmose, 2002). In this interaction a group of teachers are exchanging their views on a mathematical activity to which they have been exposed. In the interaction is also involved a teacher educator, who is in charge of guiding and moderating the discussion. It will be shown that the asynchronous interaction analyzed possesses some of the communicative characteristics of a dialogue; second, it will be argued that the interaction can be regarded as "rich" because the sort of reflections that appear in it; and third, it will be also shown that the involvement of a teacher educator in the interaction can be a decisive factor in maintaining the dialogue.

Methodology

All the details on the methodology implemented to generate the data will be listed in this section of writing. Different aspects of the production and collection of data such as the mathematical activity applied, the selected population, the collection and presentation of the data are mentioned here.

The data that used in this study were taken from one of the Master's courses of the Prome-Cicata program. The course was taught between March and April 2008. The course was an introduction to the teaching and learning of mathematical modeling, and its aim was to reflect on the modeling process, its relevance in the teaching and learning of mathematics, and its potential implementation difficulties. The author of this paper participated in this course as a designer and coordinator, as well as a teacher educator.

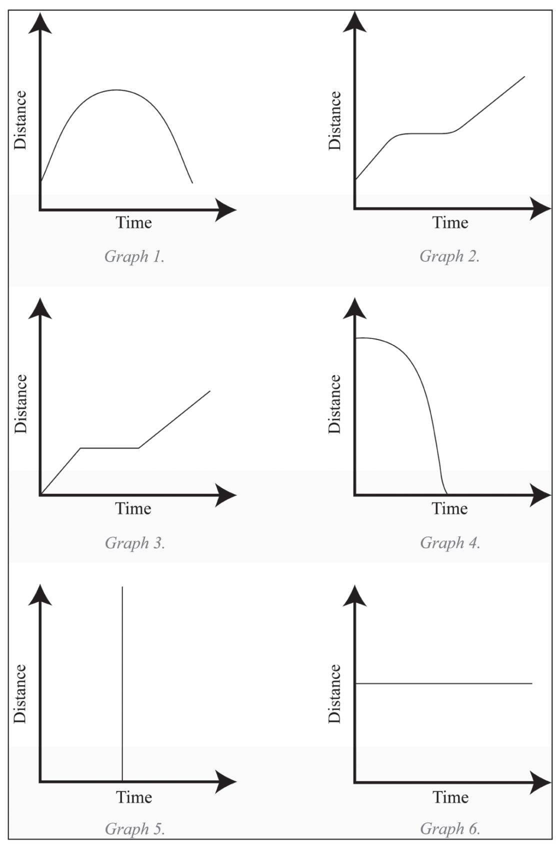
The mathematical activity

Several mathematical tasks were designed to try to meet the aims of the course. The first of those activities, called *A1*, was aimed at illustrating the possibility of providing mathematics students with modeling activities —in this case supported by the use of technology— that allow them to acquire, at least in an intuitive way, some mathematical concepts or notions. In the particular case of the activity *A1*, it refers to the possibility of connecting notions such as velocity or acceleration to the shapes of a graph in the Cartesian coordinate system, which represents the movement of a body over time.

The activity *A1* has a *note of reflection* format, that is to say, it is a written case of a fictitious classroom event arranged around a mathematical question or activity. Although this is a school episode that has not happened in reality, the answers that the "imaginary" students provide have been inspired by real teaching experiences or have been taken from experimental data included in some regional research thesis or research reports —see Sánchez, en prensa, for a more detailed description and discussion of this sort of didactical design.

The activity shows a set of six graphs (figure 1) that a teacher showed to her students after they watch the video called *V1*. This video shows a person who is illustrating how a motion sensor, connected to a graphic calculator, produces graphs which represent the movement of a body.[2]

Figure 1
Graphs included in the mathematical activity applied.



Source: Own elaboration.

2 It is important to watch the video *V1* in order to fully understand the context in which the interaction between teachers takes place. Video *V1* is available at https://bscw.ruc.dk/pub/bscw.cgi/21056220

After watching this video, the teacher asked her imaginary students —called Chuy and Mauricio— how should they move themselves in order to be able to produce with the motion sensor each of the six graphs that she was showing them. The responses of the students are included in the note of reflection. In turn, in-service teachers should watch the video *V1*, and then to evaluate the responses made by Chuy and Mauricio, in other words, they had to decide whether the answers were correct and to argue why. Teachers should send their evaluations by email to the coordinator of the course.

Some of the graphs included in the activity *A1* can be difficult to interpret. Studies such as the one conducted by Dolores, Alarcón and Albarrán (2002), reported that the assignment of a physical meaning to graphs such as the number 5 (figure 1) can be a complicated task for some mathematics students and even for teachers. This sort of graphs were included in the activity to meet one of the implicit aims of the note of reflection, namely, locating the teachers with difficulties in reading or interpreting graphs of the type time-distance that represents the motion of a body over time. Asking the teachers to evaluate the responses of Chuy and Mauricio was an indirect way of knowing their interpretations about the kind of movement represented in the graphs.

When teacher's evaluations were received by email, they were classified according to their responses and afterwards some heterogeneous working groups were constituted, i.e. working groups where the members had different views about how the graphs could be produced by using the motion sensor. Those heterogeneous working groups were set up to try to promote discussion and interaction: opinion heterogeneity has been pointed out as a contributing factor to the computer mediated dialogue (de Vries, Lund & Baker, 2002; and McGraw *et al*. 2007).

Selected population

Each working group discussed the content of the activity *A1* in an asynchronous forum that lasted six days. The discussion within each forum —there was a forum for each working group— was moderated and guided by a teacher educator. All the teachers who participated in the activity did it willingly, because they were informed at the beginning of the course that this activity was not part of their final grade for the course.

Some groups were tracked. Those groups where at least one teacher with difficulties in interpreting some of the Cartesian graphs, expressed his or her opinion or doubts about this issue in the discussion forum. The analysis was then focused on observing the reaction of their peers and the development of the asynchronous discussion. In this writing it will be presented only the analysis of the interaction within one of those groups. In the interaction three teachers and one teacher educator were involved: Alberto who is a Mexican teacher teaching mathematics at upper and lower secondary level; Susana is an Argentinean mathematics teacher who works in upper secondary and University level; Mariana who is also a teacher from Argentina who works in upper secondary level and is also a teacher educator in her home country; and last but not least, Graciela the teacher educator assigned to this group, she is a young mathematics educator researcher who recently has integrated to the teacher educators staff of the Prome-Cicata.

Data collecting and data presentation

One of the characteristics of the computer mediated communication is that it can be easily recorded, stored and shared. This feature represents a significant advantage for educational research, because the need of making transcriptions disappears. In this work for instance, are being studied some of the written asynchronous discussions produced in an internet-based educational program; those discussions are permanently recorded and accessible on the internet-based workspace, ready to be analyzed.

These asynchronous discussions may be composed of dozens of utterances. Due to space reasons, it will not be possible to present the complete interaction, but only those sections of it that are considered most significant and illustrative. It will be used bracketed ellipsis [...] to denote the omission of certain segments of text; this edition was made for the sake of brevity and to increase the readability of the data. The data presented has been translated from Spanish into English; moreover, the original names of the teachers and the teacher educator involved in the interaction have been replaced to protect their identity. During the application of the IC-Model to the analysis of the interaction, each of the utterances that constitute the interaction have been labeled with the names of the communicative acts that define the IC-Model. To facilitate its identification, those labels are written using italics.

Results

As it was expected, the graph that proved to be more difficult to interpret was the graph number 5. This is a graphical representation of a functional relationship that is not possible to translate into physical or mathematical terms. In physical terms it would be necessary to have a body occupying different positions in space in a single instant of time. In mathematical terms one can argue that the graph 5 can not represent a real function in one variable, because there is an element of the domain of the function which corresponds with more than one element of the codomain.

In the note of reflection, the 'imaginary' students Chuy y Mauricio said that this was the way in which a person should move to produce such graph: *Chuy: In graph number 5 the person should move away from the wall with a constant time. Mauricio: In the fifth graph the person should change his position with an infinite velocity.*

The sixth graph also caused some difficulties. In mathematical terms, this graph represents a constant function where $f(x) > 0$ for all x in the domain of the function. In the context were the video *V1* takes place, this graph could be generated by standing a little away from the wall, and holding the motion sensor without changing position or making any movement. Chuy and Mauricio think that the sixth graph can be generated in this way: *Chuy: In the sixth graph the movement should be horizontal in order to have the same distance, but keeping the time running. Mauricio: In the last graph, a person walks toward a wall and just before reaching it, the person turns to the right and walks with a variable time and a constant distance.*

It is important to remember that the answers of these imaginary students were originally produced by real mathematics in-service teachers, who were previously confronted with these graphs. Those teachers did not participate in this course. In turn, Alberto in his individual assessment of the responses of Chuy and Mauricio said the following:

> *Regarding graph 5: Chuy is right when he says that the person is moving away, but not when he says with a constant time. On the other hand Mauricio talks about an 'infinite velocity', maybe he means that he does it very fast and this coincide with the graph.*
> *Regarding graph 6: Both notice that there is a movement, but the graph only shows the person "unmoving" and the time running (without moving).*

Apparently, Alberto thinks that it is possible to produce the graph number 5, however, from the previous quote is not possible to determine what he thinks about how the person should move to produce it. The analysis that it will be shown next it focuses in the moment when Alberto exposes his previously mentioned interpretations in the asynchronous discussion forum. The analysis also includes the reaction of his colleagues to these comments.

The interaction analysis

When the first discussion forum started, the first discussion topic was the one introduced by Alberto. He repeated the answers from his individual solution to the activity, but adding some comments to it:

> *(1) Topic: The first contribution*
> *From: Alberto*
> *Date: thursday, the 27th of march 2008, 10:05*
> *[...]*
> *Graph 5. Chuy is right by saying that the person is moving away, but not when he says with a constant time. On the other hand MAURICIO talks about an "infinite velocity", maybe he means that he does it very fast and this coincide with the graph. Did he jump?*
> *Graph 6. Chuy and Mauricio notice that there is a movement, but the graph only shows the person "unmoving" and the time running (without doing any movement).*
> *I will wait for your comments that always are so valuable, to enrich this forum.*
> *Best regards!*
> *Alberto*

As mentioned before, it seems that Alberto thinks that is possible to produce the graph number 5 if a person moves very quickly or jumps; but is not possible to physically produce this graph using the motion sensor, neither is mathematically coherent. Graph number 5 cannot represent a mathematical function. To express our ideas and beliefs about a topic in an open way —as Alberto does— is regarded as a *thinking aloud* communicative act. The first reaction to Alberto's comment was produced by Susana. She did not agree with Alberto's ideas:

> *(2) Topic: Re: The first contribution*
> *From: Susana*
> *Date: thursday, the 27th of march 2008, 12:27*
> *[...]*
> *Graph 5. Here you will notice that I disagree with you Alberto because the explanation given by Chuy sets up an impossible situation, because is not possible for a person to be in different places at the same instant t, I mean to be away and close from the wall at the same time. Is not a mathematical function, and it does not make sense physically. […]*
> *Graph 6. In this last graph, Chuy does not consider the person without doing any movement, thus when time runs the distance does not change, because the person is located at certain distance from the wall, and because there is no movement the motion sensor does not register any variation. What I did not understand is when Mauricio says "just before reaching it, the person turns to the right and walks with a variable time and a constant distance", nevertheless it is valid to think it as walking in a parallel way to the wall even though the sensitivity of the motion sensor will show some variation.*
> *If some of you can explain me Mauricio's comment I will be grateful because I don't understand.*
> *[…]*
> *Susana*

In (2) Susana is *getting in contact* with Alberto, that is to say, she makes explicit reference to Alberto's comments and she makes some remarks about it. In fact some of these remarks could be viewed as an *evaluative* act, because she points out and explains why she does not agree with Alberto's interpretation of graph number five. After Susana's participation, Mariana joined the discussion:

> *(3) Topic: Re: The first contribution*
> *From: Mariana*

Date: thursday, the 27th of march 2008, 15:47
Hello Alberto and Susana…
I have read your comments regarding graph 5, in my opinion both students don't give the right answer, this position is similar to your answer Susana. The idea of giving a big jump will not be represented by a vertical segment; in this case it would have a negative slope, with an angle very close to a right angle but never perpendicular to the X axis. Besides we can ask what does Mauricio means with an infinite velocity? Infinitely fast or infinitely slow?
[…]
Mariana

4) Topic: Re: The first contribution
From: Mariana
Date: thursday, the 27th of march 2008, 15:55
Colleagues, another comment, but now regarding the last graph.
It is true, the faster answer is to say that there is no movement but we cannot say it categorically, with the imposed conditions we can only say that the sensor does not register any movement having the wall as a reference.
According to the graph it is possible to think in two options, the person stays without movement in a place away from the wall or he walks the distance in a parallel way but taking care of keeping the motion sensor focused on the wall and not on the place where he is walking to, but in reality this walk will produce a small disruption in the graph generated by the sensor, but in theory we can accept this possibility by ignoring external modifications, it is possible that the subject keeps the balance and walks exactly in a straight line, always keeping the sensor focused on the wall.
Mariana

In her utterances (3) and (4) Mariana is also *getting in contact* with Alberto and Susana. In an *evaluative* act, she rejected the idea of the jump suggested by Alberto for the graph number 5, and in (4) she accepts as valid the two physical interpretations that have been made for the sixth graph.

It is important to note that so far, Susana and Mariana have *kept the contact* with Alberto by 'listening' and analyzing his comments. Both teachers have shown, through *evaluative* acts, the reasons why they do not agree with the initial stance of Alberto where he said that Chuy's interpretation of graph 5 was correct (utterance 1). It could be argued that this disposition to listen, to analyze and to evaluate Alberto's ideas is an indicator that these teachers have established a dialogue and, that the comments offered by Susana and Mariana, constituted a good reflection opportunity for Alberto; an opportunity to review his own comments and ideas and to try to verify its validity. It is possible to confirm this last assertion by looking at Alberto's reaction to those evaluative acts.

In another discussion topic that he started in the same discussion forum, Alberto said:

(5) Topic: Graph 5
From: Alberto
Date: thursday, the 27th of march 2008, 19:19
Hello everybody
Reflecting on graph 5, it does not make sense physically… and theoretically it would be impossible. We can see that the slope of the straight line is indefinite, because it reaches a value of 90º.
Taking the slope formula as velocity for this graphs, distance versus time, we have V=m=(d2—d1)/(t2—t1). Graphically we can see that there is a displacement, but time doesn't change, it is the same, so: t2—t1=0. Carrying out the division, we have that: V=m=(d2—d1)/0, and this is indefinite.
Therefore, I think there is no a behavior with the "motion sensor" that could produce a graph like this one.
What do you think colleagues?
Best regards
Alberto

Stimulated by the evaluative acts of Susana and Mariana, Alberto somehow changed his mind regarding graph 5. Probably Alberto *identified* the mathematical structure of the situation, noticing the impossibility of producing a function whose graphical representation is like the one presented in graph number 5. Interestingly, Alberto is not the only one who seems to have discovered something new; Susana in (6) is locating another way to justify in mathematical terms that the graph number 5 is not possible to produce:

(6) Topic: Re: Graph 5
From: Susana
Date: thursday, the 27th of march 2008, 21:48
Alberto: Of course I agree with your way of analyzing the situation, I had not thought it from a theoretical point of view taking into account the concept of average velocity and the variations of time and distance. This because I thought that using the concept of mathematical function would be enough, because it is not a function since for a t value you have more than one ordinate value.
What do you think colleagues? As always your comments and different point of views are welcome.
[...]
Susana

So far, this interaction could be considered as a rich one. Alberto started the discussion in (1) evidencing a not accurate physical interpretation of the graph number 5; Alberto fortunately got the attention and criticism of his colleagues that, according to my interpretation, it helped Alberto to revise his initial idea and express an adjustment to it in (5). Although the intervention number (5) suggested that Alberto has understood that it is not possible to consider graph number 5 as a representation of a real function in one variable, his interpretation of the sixth graph was not free of difficulties. In an earlier statement, Alberto used the formula $v = d/t$ as a basis for *evaluating* one of the ideas put forward by Susana in (2) with respect to the sixth graph:

(7) Topic: Re: The first contribution
From: Alberto
Date: thursday, the 27th of march 2008, 18:58
Hello Susana
Regarding graph 6, you said "it is valid to think it as walking in a parallel way to the wall even though the sensitivity of the motion sensor will show some variation".
I think your idea is not valid, because we can see that the slope of the straight line is equal to zero, as well as its velocity. Let's suppose that he doesn't move. Then in a time t the person has a distance d=0; and for V=d/t we will have V=0/t, and the result will be V=0.
Now, taking your idea and supposing that there is a displacement of the person (parallel to the wall), the velocity (V=d/t) would be different from zero and the slope would have certain inclination although small.
What do you think colleagues?
Best regards!
Alberto

Even though Alberto's comment was explicitly directed to Susana, Mariana *gets in contact* with Alberto and *challenge* him with regard to his argumentation, in other words, she is trying to show him that there is an alternative way to interpret the graph:

(8) Topic: Re: The first contribution
From: Mariana
Date: thursday, the 27th of march 2008, 19:29
Alberto: first of all in graph 6 distance is not zero but constant, thus the two options are valid: to stay without movement or to walk in a parallel way to the wall.
The motion sensor registers, let's say it like this, the movement between to walls of a room, but it doesn't register subject's movements towards other directions like the lateral walls. If the person could levitate he could move towards the roof and the motion sensor will register a constant distance. We can expect that the sensor will register some variations in the case of walking laterally, but it depends on its sensitivity.
I don't know what Susana thinks, because I meddle in this comment.
Mariana

Susana also contributes to the discussion about the interpretation of the sixth graph by *getting in contact* with Mariana and Alberto, but particularly by *evaluating* Alberto's argument:

(9) Topic: Re: The first contribution
From: Susana
Date: thursday, the 27th of march 2008, 21:37
Hello Alberto, as Mariana said (there is no problem with your intervention) distance is not zero, but the variation in the position is zero -distance is constant-.
You are right when you think that the average velocity of the movement is the quotient of position variation by time[.] [B]ecause that variation is zero over time then the average velocity is zero, meaning that there was no movement.
The other option of having a parallel displacement is weird but you are right by saying that it would produce a slope in the straight line [...]
Is it my answer clear? [...]
Susana

A plausible interpretation here is that both Susan and Mariana accepted that the sixth graph can be generated by having a person standing in front of a wall, without doing any movement but focusing the motion sensor to a fixed point on the wall. However, they also have seen another way to produce the same graph: by performing a parallel movement to the wall where the sensor is aiming at. Apparently, this second possibility is not so obvious to Alberto who in (10) attempts to *reformulate* his position, that is, to repeat what he has just said but maybe in slightly different words (Alrø & Skovsmose, 2002, p. 108):

(10) Topic: Re: The first contribution
From: Alberto
Date: thursday, the 27th of march 2008, 23:04
Susana, Moni, colleagues…
Being in a coordinate system (as I stressed) distance versus time, obviously I'm talking about the distance that the person covers as time goes by.
To avoid confusion, I will call displacement to the movement that a person does (vertical axis) and I will set out again the idea:
When time is running the displacement is the same, in other words, the person does not move and, therefore, his velocity is zero because the slope is also equal to zero. That is to say, in a time t, person's displacement is d=0 , then taking the formula V=d/t we will have V=0/t, and then V=0.
I hope I have clarified the ambiguity, I wish you a pleasant friday.
Alberto

One could say that at this point in the interaction, Alberto has not located the other way to interpret the graph 6 that his colleagues Susana and Mariana have *located*, namely, that at least hypothetically and in the context in which the video *V1* takes place, it would be possible to make a movevement parallel to the wall to produce such a graph. One hypothesis, based on previous observations of interactions between in-service teachers (Sánchez, 2008), is that the communicative acts such as evaluating and challenging in a dialogue, are elements that promote reflection and the revision of ideas of whom is being evaluated and challenged. If Susana and Mariana had continued the dialogue with Alberto it may have helped him to locate the alternative interpretation of the sixth graph, but as will be shown right away, the participation of a teacher educator in the interaction can be a determinant factor in maintaining a dialogue.

Graciela, the teacher educator in charge of coordinating the interaction within this group of teachers, contributed to the discussion of the interpretation of the graphs with the following comment:

> *(11) Topic: The rol of technology*
> *From: Graciela*
> *Date: friday, the 28th of march 2008, 09:55*
> *Everybody's attention is attracted by the physical impossibility (real) of graph 5. Nevertheless we can "force" technology (particularly these sensors and also the calculators) to produce a vertical line. Something similar will happen with discontinuous functions. This makes me ask for your opinion about the role of technology in this modeling process. If we consider that from reality we go to a model and then to an analysis, what is the role of technology?*

After this remark teacher's attention and the discussion itself was redirected, that is, the dialogue between Susana, Mariana and Alberto was interrupted in order to meet the new topic proposed by the teacher educator, namely, the role of technology in a mathematical modeling process.

Probably at the time of her participation in (11), Graciela did not prevent that her participation could become a kind of disruption to the dialogical interaction that had emerged among the teachers. In fact, Graciela's behavior could be better understood if a broader context of analysis is considered: before and after each forum, the teachers educators who participated in this course talked and exchanged their impressions —using email or internet-based audio communication— on the aims or purposes that should be pursued by each forum and activity. In one of the emails that was sent to the teachers educators who participated in the course (including Graciela), it was suggested that some of the aspects that could be addressed during this forum might be:

> *We can make some critiques to the sort of modeling activity described in A1 (using technology, analyzing the graphical representations of displacement vs. time), for example:*
> *If we agree that in general, a modeling process is a cycle having the form reality-model-analysis and results-reality. What is the role that this sort of [technological] devices plays in this modeling process?*

Thus, although one could argue that the participation of Graciela was guided or motivated by the goals of the forum previously agreed by the teachers educators, what is relevant to emphasize here is how fragile the permanence of a dialogue can be and the considerable influence that the authority of the teacher educator can exert in the conservation of a dialogue.

Conclusions

The interaction in which Susana, Mariana and Alberto participated can be viewed as a dialogue, as defined in Alrø & Skovsmose (2002), as the participants of the interaction were willing to express in public their ideas on the mathematical activity, and also were continuously paying attention to the ideas of the others but also evaluating and criticizing them, in an environment modulated by tolerance and respect.

When the participants of an interaction are able to establish a *dialogue*, it can serve as a basis for the emergence of rich reflections that can provide the participants with opportunities to identify and review their ideas and conceptions, and in some cases, to modify them in a positive way. This was the case of Alberto, who was driven by the evaluative and challenging acts of Susana and Mariana, and apparently changed his initial conception on the graph number 5 understanding that the graph did not make sense in a mathematical context (utterance number 5). In fact one of the hypothesis that emerge from this analysis, and that match previous findings (Sanchez, 2008), is that the communicative acts *evaluating* and *challenging* play a key role in a dialogue.

It is claimed that these communicative acts are elements that are necessary not only for the validation of our ideas and thoughts, but they also contribute to its consolidation and development, they may even give rise to new ideas or to a modification of the original ideas.

Another important point in this writing is the potential impact of the teacher educator in the permanence of a dialogue. Teacher educators should be aware that very often, their interventions in an interaction which teachers are burdened with an implicit authority assigned by the teachers. Teachers pay particular attention to the comments, ideas, proposals and criticism posted by teacher educators and often this attention is bigger than the one that the teachers pay to their fellow teachers' comments.

It is possible to find an explanation for this phenomenon if is accepted that there is a kind of didactical contract among teacher educators and teachers, where the teacher educator holds a status of expert and authority that teachers recognized as such. This raises an asymmetrical relationship between teachers and teacher educators that can be an obstacle to the establishment and permanence of a dialogue because, as claimed by Alrø & Skovsmose (2002, p. 124): *A dialogue is based on the principle of equality […] A dialogue cannot be modulated by the roles (and the power associated with these roles) of the persons participanting in the dialogue*.

The previously presented data has illustrated how the teachers are leaving the dialogue established with Alberto, just to follow the course of the discussion framed by Graciela. This is an example of how inequality —to prioritize the ideas of one of the participants in the interaction— can lead to a breakdown in a dialogue.

However, is not possible to deny that the relationship between teachers and teacher educators is somewhat unequal. The knowledge that both groups possess are different and the interest to learn and to share that knowledge is what gives meaning to the academic relationship teacher-teacher educator, but how then maintain equality in such an asymmetrical relationship? According to Rogers (1962, 1994), quoted in Alrø & Skovsmose (2002, pp. 125-126), particular qualities of contact are important in order to maintain equality in an asymmetrical relationship: congruence, empathy and positive regard. *Being congruent means being genuine without any front or facade. The facilitator's thoughts and feelings should be consistent with his way of acting, and this should be obvious to him or herself and to the other person. Congruence stands for transparency and genuineness […] Empathy means that the facilitator tries to understand the other's person's world as if were his or her own […] The third condition is positive regard. In order to be able to help another person you have to accept and to respect him or her and as a(nother) person. This implies respecting the otherness of the other without intending to change him or her as a person*.

It is necessary for teachers and teacher educators to be aware that the quality of communication between them has implications on the quality of knowledge and professional development that they get through this communication. In this work, a set of interpretations and hypotheses was presented with the aim of initiating a dialogue with teachers and teacher educators, about the possible routes that could be explored to try to improve the quality of the mathematical education that is offered and received, and that hopefully will be positively reflected in the quality of education that the students receive in the mathematics classroom.

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