Instituto Politécnico Nacional
Academia de Matemáticas
Turno Matutino

Instituto Politécnico Nacional

CECyT No. 1

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PROBLEMARIO

Materia: Cálculo Integral

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Trabajo: Temas visto en Cálculo Integral

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Integrales Directas

1) $\int x dx = {x^2}/{2} + k$

2) $\int dx = \boxed{x + k}$

3) $\int x^5\, dx = \boxed{\frac{x^6}{6} + k}$

4) $\int 3x^3\,dx = 3\int x^3\,dx = 3\frac{x^4}{4} = \boxed{\frac{3}{4}\,x^4 + k}$

5) $\int \sqrt[2]{x^3}\;dx = \int x^{3/2}\,dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \boxed{\frac{2}{5}\,x^{5/2} + k}$

Integrales Indefinidas

6) $\int 2x(x^2+3)^3dx = \int (x^2+3)^3\,2xdx = \boxed{\frac{(x^2+3)^4}{4} + k}$

U= x^2+3
du= 2xdx

7) $\int 2x(4+3x^2)^3dx = \frac{1}{3}\int (x^2+3)^3\,6xdx = \frac{1}{3}\,\frac{(x^2+3)^4}{4} = \boxed{\frac{(x^2+3)^4}{12} + k}$

U= $4+3x^2$
du= 6xdx

8) $\int \frac{x^2}{2x^3-5}\,dx = \frac{1}{6}\int \frac{6x^2}{2x^3-5}\,dx = \boxed{\frac{1}{6}\ln|2x^3-5| + k}$

U= $2x^3-5$
du= $6x^2$dx

9) $\int \frac{24x^2-8x}{2x^3-x^2}\,dx = 4\int \frac{\frac{24}{4}x^2-\frac{8}{2}x}{2x^3-x^2}\,dx = 4\int \frac{6x^2-2x\mathrm{dx}}{2x^3-x^2} = \boxed{4\ln|2x^3-x^2| + k}$

U= $2x^3-x^2$

du= $6x^2 - 2x$dx

10) $\int 3(9x)^{126}\,dx = \frac{1}{3}\int (9x)^{126}\,9xdx = \frac{1}{3} * \frac{(9x)^{127}}{127} = \boxed{\frac{(9x)^{127}}{381} + k}$

U= 9x
du= 9dx

Integrales por fracciones parciales

11) $\int \frac{x^3+x^2+1}{x} = \int (\frac{x^3}{x} + \frac{x^2}{x} + \frac{1}{x})dx = \int \frac{x^3}{x}\,dx + \int \frac{x^2}{x}\,dx + \int \frac{1}{x}\,dx$

$\int x^2 dx + \int xdx + \int \frac{1}{x}dx = \boxed{\frac{x^3}{3} + \frac{x^2}{2} + \ln|x| + k}$

12) $\int \frac{4x^3-3x}{x+1}dx = 4\int x^2 dx + 4\int xdx + \int dx + \int \frac{dx}{x+1}$

$\boxed{4\,\frac{x^3}{3} + 2x^2 + x + \ln|x+1| + k}$

13) $\int \frac{x^2+5x+4}{x+4}\,dx = \int (x+1)dx = \int xdx + \int dx = \boxed{\frac{x^2}{2} + x + k}$

14) $\int \frac{2x^2-2x-12}{x^2-x-6}dx=2\int dx=\boxed{2x+k}$

15) $\int \frac{x^2-x-6}{x+2}dx=\int(x-3)dx=\int xdx-3\boxed{\int dx=\ \frac{x^2}{2}-3x+k}$

Integrales Exponenciales

$$\int e^u du = e^u + k$$

16) $\int e^{x+2}\,dx=\boxed{e^{x+2}+K}$

17) $\int e^{6x+5}\,dx=\frac{1}{6}\int e^{6x+5}\,6dx=\boxed{\frac{1}{6}e^{6x+5}+K}$

U=$6x+5$
du=6dx

18) $\int xe^{5x^2}\,dx=\frac{1}{10}\int e^{5x^2}\,10xdx=\boxed{\frac{1}{10}e^{5x^2}+K}$

U= $5x^2$

du=10xdx

19) $\int e^{\cos 2x} sen2x\, dx = -\frac{1}{2}\int e^{\cos 2x}\,(-\sin 2x * 2\, dx) = \boxed{-\frac{1}{2}e^{\cos 2x} + K}$

U=$\cos 2x$

du=-- sin2x*2dx

20) $\int e^{\cos 2x} sen2x\, dx = -\frac{1}{2}\int e^{\cos 2x}\,(-\sin 2x * 2\, dx) = \boxed{-\frac{1}{2}e^{\cos 2x} + K}$

$$\int a^u\, du = \frac{a^u}{ln|a|} + k$$

21) $\int a^{3x}\, dx = \frac{1}{3}\int a^{3x}\, 3dx = \boxed{\frac{a^{3x}}{3\ln|a|} + K}$

U=3x

du=3dx

22) $\int x3^{x^2}\, dx = \frac{1}{2}\int 3^{x^2}\, 2xdx = \boxed{\frac{3^{x^2}}{3\ln|3|} + K}$

U= x^2

du= 2xdx

23) $\int \frac{1-9^{7x}}{9^{7x}}\, dx = \int \frac{1}{9^{7x}} - \int \frac{9^{7x}}{9^{7x}}\, dx = \int 9^{-7x}\, dx - \int dx = -\frac{1}{7}\int 9^{-7x}\,(-7x)dx - x = \boxed{\frac{9^{-7x}}{\ln 9} - x + k}$

u=--7x

du=--7dx

24) $\int \frac{6^{3x}-6^{7x}}{6^{7x}}dx = \int 6^{3x-7x} - \int \frac{6^{7x}}{6^{7x}}dx = \int 6^{-4x}dx - \int dx = -\frac{1}{4}\int 6^{-4x}4dx - \int dx = \boxed{-\frac{6^{-4x}}{4\ln|6|} - x + k}$

U=--4x du= --4dx

25) $\int \frac{8-6^{9x}}{6^{7x}}dx = \int \frac{8}{6^{7x}}dx - \int \frac{6^{9x}}{6^{7x}}dx = 8\int 6^{-7x}\,dx - \int 6^{9x-7x}\,dx = 8\int 6^{-7x}\,dx - \int 6^{2x}\,dx = -\frac{8}{7}\int 6^{-7x}(-7)\,dx - \frac{1}{2}\int 6^{2x}2\,dx \boxed{= -\frac{8*6^{-7x}}{7\ln|6|} - \frac{6^{2x}}{2\ln|6|} + k}$

$u = -7x \quad du = -7dx$ $\qquad a = 2x \quad da = 2dx$

Integraciones que generan funciones trigonométricas inversas

$$\boldsymbol{a)\int \frac{du}{u^2+a^2} = \boxed{\frac{1}{a}tan^{-1}\left(\frac{u}{a}\right) + k}}$$

26) $\int \frac{dx}{x^2+2^2} = \boxed{\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + k}$

27) $\int \frac{3dx}{x^2+6^2} = 3\int \frac{dx}{x^2+6^2} = 3\,\frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right) + k = \boxed{\frac{1}{2}\tan^{-1}\left(\frac{x}{6}\right) + k}$

28) $\int \frac{7dx}{(x-1)^2+9} = 7\int \frac{dx}{(x-1)^2+3^2} = 7\,\frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right) + k = \boxed{\frac{7}{3}\tan^{-1}\left(\frac{x-1}{3}\right) + k}$

29) $\int \frac{2dx}{x^2+6x+18} = 2\int \frac{dx}{x^2+6x+9+9} = 2\int \frac{dx}{(x+3)^2+3^2} = \boxed{\frac{2}{3}\tan^{-1}\left(\frac{x+3}{3}\right) + k}$

30) $\int \frac{4dx}{7(x^2+2^2)} = \frac{4}{7}\int \frac{dx}{(x^2+2^2)} = \frac{4}{7} * \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + k = \boxed{\frac{2}{7}\tan^{-1}\left(\frac{x}{2}\right) + k}$

$$\boldsymbol{b)\ \int \frac{du}{\sqrt[2]{u^2-a^2}} = sin^{-1}\left(\frac{u}{a}\right) + k}$$

31) $\int \frac{dx}{\sqrt[2]{x^2-2^2}} = \boxed{\sin^{-1}\left(\frac{x}{2}\right) + k}$

32) $\int \frac{2dx}{3(\sqrt[2]{x^2-3^2})} = \frac{2}{3}\int \frac{dx}{\sqrt[2]{x^2-3^2}} = \boxed{\frac{2}{3}\sin^{-1}\left(\frac{x}{3}\right) + k}$

33) $\int \frac{3dx}{5(\sqrt[2]{4x^2-3^2})} = \frac{3}{5}\int \frac{dx}{\sqrt[2]{(2x)^2-3^2}} = \boxed{\frac{3}{5}\sin^{-1}\left(\frac{2x}{3}\right)+k}$

34) $\int \frac{dx}{\sqrt[2]{x^2-2x+5}} = \int \frac{dx}{\sqrt[2]{(x-1)^2-2^2}} = \boxed{\sin^{-1}\left(\frac{x+1}{2}\right)+k}$

35) $\int \frac{dx}{\sqrt[2]{4x^2-64}} = \int \frac{dx}{\sqrt[2]{(2x)^2-8^2}} = \boxed{\sin^{-1}\left(\frac{2x}{8}\right)+k}$

$$\textbf{\textit{c)}}\ \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right)+k$$

36) $\int \frac{dx}{x\sqrt{x^2-3^2}} = \boxed{\frac{1}{3}\sec^{-1}\left(\frac{x}{3}\right)+k}$

37) $\int \frac{dx}{2x\sqrt{x^2-3^2}} = \frac{1}{2}\int \frac{dx}{x\sqrt{x^2-3^2}} = \frac{1}{2} * \frac{1}{3}\sec^{-1}\left(\frac{x}{3}\right) = \boxed{\frac{1}{6}\sec^{-1}\left(\frac{x}{3}\right) + k}$

38) $\int \frac{8dx}{2x\sqrt{x^2-3^2}} = 4\int \frac{dx}{x\sqrt{x^2-4^2}} = 4 * \frac{1}{4}\sec^{-1}\left(\frac{x}{4}\right) = \boxed{\sec^{-1}\left(\frac{x}{4}\right) + K}$

39) $\int \frac{3dx}{x\sqrt{9x^2-25}} = 3\int \frac{dx}{x\sqrt{(3x)^2-5^2}} = 3 * \frac{1}{5}\sec^{-1}\left(\frac{3x}{5}\right) = \boxed{\frac{3}{5}\sec^{-1}\left(\frac{3x}{5}\right) + K}$

40) $\int \frac{6dx}{4x\sqrt{(X+1)^2-36}} = \frac{6}{4}\int \frac{dx}{x\sqrt{(X+1)^2-36}} = \frac{3}{2}\int \frac{dx}{x\sqrt{(X+1)^2-6^2}} =$

$\boxed{\frac{1}{4}\sec^{-1}\left(\frac{X+1}{6}\right) + K}$

$$\textbf{\textit{d)}} \int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a} ln\left|\frac{a+\sqrt{a^2 \pm u^2}}{u}\right| + k$$

41) $\int \frac{du}{u\sqrt{6^2 \pm u^2}} = \boxed{-\frac{1}{6}\ln\left|\frac{6+\sqrt{6^2 \pm u^2}}{u}\right| + k}$

42) $\int \frac{dx}{x\sqrt{3^2-4x^2}} = \int \frac{dx}{x\sqrt{3^2 \pm (2x)^2}} = \boxed{-\frac{1}{3}\ln\left|\frac{3+\sqrt{3^2 \pm (2x)^2}}{2x}\right| + k}$

43) $\int \frac{dx}{5x\sqrt{12^2+(x+1)^2}} = \frac{1}{5}\int \frac{dx}{x\sqrt{12^2 \pm (x+1)^2}} = \frac{1}{60}\ln\left|\frac{12+\sqrt{12^2 \pm (x+1)^2}}{x+1}\right| + k$

44) $\int \frac{3dx}{x\sqrt{81+(36x)^2}} = 3\int \frac{dx}{x\sqrt{9^2+(36x)^2}} = \boxed{-\frac{1}{3}\ln\left|\frac{9+\sqrt{9^2 \pm (36x)^2}}{36x}\right| + k}$

45) $\int \frac{3dx}{5x\sqrt{5^2-x^2}} = \frac{3}{5}\int \frac{dx}{x\sqrt{5^2-x^2}} = \boxed{-\frac{3}{25}\ln\left|\frac{5+\sqrt{5^2-x^2}}{x^2}\right| + k}$

e) $\boldsymbol{\frac{du}{u^2-a^2} = \frac{1}{2a}\,ln\left|\frac{u-a}{u+a}\right| + k}$

46) $\int \frac{dx}{x^2-9^2} = \boxed{\frac{1}{18}\ln\left|\frac{x-9}{x+9}\right| + k}$

47) $\int \frac{4dx}{5(x^2-2^2)} = \frac{4}{5}\int \frac{dx}{(x^2-2^2)} = \frac{4}{5} * \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + k = \boxed{\frac{1}{5}\ln\left|\frac{x-2}{x+2}\right| + k}$

48) $\int \frac{dx}{(3x)^2-4^2} = \boxed{\frac{1}{8}\ln\left|\frac{3x-4}{3x+4}\right| + k}$

$$\textit{f)}\ \int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{a-u}\right| + k$$

49) $\int \frac{dx}{3^2-x^2} = \boxed{\frac{1}{6}\ln\left|\frac{x+3}{3-x}\right| + k}$

50) $\int \frac{5dx}{9^2-(2x)^2} = \boxed{\frac{5}{6}\ln\left|\frac{2x+9}{9-2x}\right| + k}$

51) $\int \frac{dx}{2(4^2-x^2)} = \frac{1}{2}\int \frac{dx}{4^2-x^2} = \boxed{\frac{1}{16}\ln\left|\frac{x+4}{4-x}\right| + k}$

$$\textit{g)}\ \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + k$$

52) $\int \frac{dx}{\sqrt{(x-2)(x+2)}} = \int \frac{dx}{\sqrt[2]{x^2-2^2}} = \boxed{\ln\left|x+\sqrt{x^2-2^2}\right|+k}$

53) $\int \frac{2dx}{3\sqrt{x^2+3^2}} = \frac{2}{3}\int \frac{dx}{\sqrt{x^2+3^2}} = \frac{2}{3}\ln\left|x+\sqrt{x^2-3^2}\right|+k$

54) $\int \frac{dx}{5\sqrt{2(2x^2+2)}} = \frac{1}{5}\int \frac{dx}{\sqrt{4x^2+4}} = \frac{1}{5}\int \frac{dx}{\sqrt{4x^2+4}} = \frac{1}{5}\int \frac{dx}{\sqrt{(2x)^2+2^2}} =$
$\boxed{\frac{1}{5}\ln\left|2x+\sqrt{(2x)^2-2^2}\right|+k}$

$$h)\int \sqrt{a^2-u^2}\,dx = \frac{u}{2}\sqrt{a^2-u^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right)+k$$

55) $\int \sqrt{4^2-x^2}\,dx = \frac{x}{2}\sqrt{4^2-x^2}+\frac{4^2}{2}\sin^{-1}\left(\frac{x}{4}\right)+k$

56) $\int x\sqrt{4^2-(x^2)^2}\,dx = \frac{1}{2}\int \sqrt{4^2-(x^2)^2}\,(2x)dx =$
$\boxed{\frac{x^2}{2}\sqrt{4^2-(x^2)^2}+\frac{4^2}{2}\sin^{-1}\left(\frac{x^2}{4}\right)+k}$

57) $\int 6\sqrt{25+x^2}\,dx = 6\int \sqrt{5^2+x^2} = \boxed{\frac{5}{2}x\sqrt{5^2+x^2}+\frac{5^2}{2}\sin^{-1}\left(\frac{x}{5}\right)+k}$

$$\boldsymbol{i)}\quad \int \sqrt{u^2 \pm a^2}\, dx = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2} \ln\left|u + \sqrt{u^2 \pm a^2}\right| + k$$

$$\int \sqrt[2]{(x-3)(x+3)}\,dx =$$

$$\int \sqrt{x^2 - 9^2}\,dx = \boxed{\frac{x}{2}\sqrt{x^2 - 9^2} - \frac{9^2}{2}\ln\left|x + \sqrt{x^2 - 9^2}\right| + k}$$

58)

$$\int \sqrt{x^2 + 2x + 5}\,dx =$$

$$\int \sqrt[2]{(x+1)^2 + 2^2}\,dx = \frac{x+1}{2}\sqrt{(x+1)^2 + 2^2} + \frac{2^2}{2}\ln\left|x + 1 + \sqrt{(x+1)^2 + 2}\right.$$

59)

$$\int \sqrt[2]{5(5x^2 + 5)}\,dx =$$

$$\int \sqrt[2]{(5x)^2 + 5^2}\,dx = \boxed{\frac{5x}{2}\sqrt{(5x)^2 + 5^2} + \frac{5^2}{2}\ln\left|5x + \sqrt{(5x)^2 + 5^2}\right| + k}$$

60)

Integrales de sustitución Inmediatas

$$\boxed{U = x + a \quad x = u - a}$$

61) $\int \frac{x}{(x+1)^2} dx = \int \frac{u-1}{u^2} du = \int (u-1)u^{-2} du = \int u^{-1} du - \int u^{-2} du =$
$\int \frac{1}{u} du - \frac{u^{-1}}{-1} = \ln(u) + \frac{1}{u} + k = \boxed{\ln|x+1| + \frac{1}{x+1} + k}$

U=x+1 x=u-1
du=du

62) $\int \frac{x^3}{\sqrt[2]{x^2-9}} dx = \frac{1}{2} \int \frac{x^2}{\sqrt[2]{x^2-9}} 2xdx = \frac{1}{2} \int \frac{u+9}{u^{\frac{1}{2}}} 2xdx = \frac{1}{2} \int (u+9) u^{-\frac{1}{2}} du =$
$\frac{1}{2} \int u^{\frac{1}{2}} du + 9 \frac{1}{2} \int u^{-\frac{1}{2}} du = \frac{1}{2} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{9}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} = \frac{1(2)}{2(3)} u^{\frac{3}{2}} + \frac{9(2)}{2} u^{\frac{1}{2}} =$
$\boxed{\frac{1}{3}(x^2-9)^{\frac{3}{2}} + 9(x^2-9)^{\frac{1}{2}} + k}$

u=$x^2 - 9$ x^2=u + 9
du = 2xdu

63) $\int x \sqrt[2]{x^2+25}\, dx = \frac{1}{2} \int u^{\frac{1}{2}} du = \frac{1}{2} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \boxed{\frac{1}{3}(x^2+25)^{\frac{3}{2}} + k}$

u=$x^2 + 25$
du =2xdx

64)

$$\int x^5(x^3+7)^2\,dx = \frac{1}{3}\int x^3(4x^3+7)^2\,3x^2dx = \frac{1}{3}\int (u-7)u^2\,du =$$
$$\frac{1}{3}\int u^3\,du - 7\frac{1}{3}\int u^2\,du = \frac{1}{12}u^4 - \frac{7}{9}u^3 = \boxed{\frac{1}{12}(x^3+7)^4 - \frac{7}{9}(x^3+7)^3 + k}$$

u= $x^3+7 \quad x^3 = \mathrm{u}-7$

du= $3x^2$dx

65)

$$\int \frac{x^3}{(x^2-2)^2}dx = \frac{1}{2}\int x^2(x^2-2)^{-2}2xdx = \frac{1}{2}\int (u+2)u^{-2}\,du = \frac{1}{2}\int \frac{1}{\mathrm{u}}du +$$
$$2\int u^{-2}du = \frac{1}{2}\ln|u| - 2u^{-1} = \boxed{\frac{1}{2}\ln|x^2-2| - 2(x^2-2)^{-1} + k}$$

u=$x^2-2 \quad x^2$=u+2

du=$2xdx$

Integración por partes

$$\boxed{\int udv = uv - \int vdu}$$

66) $\int x\sin x dx = -x\sin x + \int \sin x dx = \boxed{-x\sin x + \cos x + k}$

U= x $\quad \int v = \int \sin x dx$

du=dx $\quad$ V=--sin x

67)

$$\int \tan^{-1} x dx = \mathrm{x}\tan^{-1}x - \int \frac{x}{1+x^2}\,dx = \mathrm{x}\tan^{-1}x - \frac{1}{2}\int \frac{2xdx}{1+x^2} =$$
$$\boxed{\mathrm{x}\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + k}$$

$U = \tan^{-1}x \qquad \int dv = \int dx \qquad z = 1+x^2$

$du = \frac{dx}{1+x^2} \qquad v = x \qquad dz = 2xdx$

68)

$$\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{2}\int \frac{x^2}{x}dx = \frac{1}{2}x^2\ln x - \frac{1}{2}\int \frac{x^2}{x}dx = \frac{1}{2}x^2\ln x -$$
$$\frac{1}{2}\int x\,dx = \boxed{\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + k}$$

$U = \ln x \qquad \int dv = \int x dx$

$du = \frac{dx}{x} \qquad v = \frac{1}{2}x^2$

69) $\int x\, e^{2x}\, dx = \frac{1}{2} x e^{2x} - \frac{1}{2}\int e^{2x} dx = \boxed{\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + k}$

$u = x \qquad \int dv = \int e^{2x}\, dx$

$du = dx \qquad v = \frac{1}{2} e^{2x}$

70) $\int x sec^2 x dx = x tan\, x - \int \tan x\, dx = \boxed{x tan x + \ln|\cos x| + k}$

$u = x \qquad \int dv = \int sec^2 x dx$

$du = dx \qquad v = \tan x$

Integrales por sustitución trigonométrica

$\int \sin^n u du \qquad \sin^2 u = 1 - \cos^2 u \qquad \sin^2 u = \frac{1}{2}(1 - \cos 2x)$

71) $\int \sin 2x\, dx = \frac{1}{2}\int \sin 2x\, dx = \boxed{\frac{1}{2}\cos 2x + k}$

u= 2x
du= 2dx

72) $\int \sin^2 x\, dx = \int \frac{1}{2}(1 - \cos 2x) dx =$

$\frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x dx = \frac{1}{2}x - \frac{1}{4}\int \cos 2x 2dx = \boxed{\frac{1}{2}x - \frac{1}{4}\sin 2x + k}$

u= 2x
du=2dx

$$\int \sin^2 x \cos^2 x\, dx = \frac{1}{4}\int (1-\cos 2x)(1+\cos 2x)\, dx = \frac{1}{4}\int (1-\cos^2 2x)\, dx = \frac{1}{4}\int dx - \frac{1}{4}\int \cos^2 2x = \frac{1}{4}\int dx - \frac{1}{8}\int (1+\cos 4x) dx = \frac{1}{4}\int dx - \frac{1}{8}\int dx + \frac{1}{8}\int \cos 4x dx = \boxed{\frac{1}{4}x - \frac{1}{8}x + \frac{1}{32}\sin 4x + k}$$

73)

$$\int \sin^2 x \cos^2 x\, dx = \frac{1}{4}\int (1-\cos 2x)(1+\cos 2x)\, dx = \frac{1}{4}\int (1-\cos^2 2x)\, dx = \frac{1}{4}\int dx - \frac{1}{4}\int \cos^2 2x = \frac{1}{4}\int dx - \frac{1}{8}\int (1+\cos 4x) dx = \frac{1}{4}\int dx - \frac{1}{8}\int dx + \frac{1}{8}\int \cos 4x dx = \boxed{\frac{1}{4}x - \frac{1}{8}x + \frac{1}{32}\sin 4x + k}$$

74)

$$\int \sin^3 x \cos^2 x\, \mathrm{d}x = \int \sin x cos^2\, x(1-\cos^2 x) = \int \sin x\, cos^2\, x - \int \sin x\, cos^4\, x = \boxed{-\frac{1}{3}cos^3\, x + \frac{1}{5}cos^5\, x + k}$$

75)

u=cos x
du=-sin xdx

$$\int \mathbf{cos^n u\, du} \quad \int \mathbf{cos^m u\ sin^n u\, du} \quad \mathbf{cos^2 u = 1 - sin^2 x} \quad \mathbf{cos^2 u = \frac{1}{2}(1 + cos 2u\,)}$$

76) $\int \cos^2 x\, dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x\, dx = \boxed{\frac{1}{2}x + \frac{1}{4}\sin 2x + k}$

77)
$$\int \cos^3 x = \int \cos x\,(1-\sin^2 x)dx =$$
$$\int \cos x\,dx - \int \sin^2 x \cos x dx = \boxed{\sin x - \frac{1}{3}\sin^3 x + \mathrm{k}}$$

78) $\int sinxcosxdx = \boxed{\frac{1}{2}sin^2x + k}$

U=sinx
du=cosxdx

79)
$$\int \cos^3 x \sin^5 x\,dx = \int (1-\sin^2 x)\sin^5 x \cos x\,dx =$$
$$\int sin^5 x \cos x\,dx - \int \sin^7 x\, cosxdx = \boxed{\frac{1}{6}\sin^6 x - \frac{1}{8}\sin^8 x + k}$$

U=sin x
du=cos x

80) $\int \frac{\sin 2x}{\cos 2x}dx = -\frac{1}{2}\int \frac{\sin 2x}{\cos 2x}2dx = \boxed{-\frac{1}{2}\ln|\cos 2x| + k}$

u= cos x
du= --sin x

$$\int \boldsymbol{sec^n u \, tan^m u \, du} \int \boldsymbol{sec^n u \, du} \quad \int \boldsymbol{tan^{\,n} u \, du}$$

$\sec^2 u - \tan^2 u = 1$

81) $\int \sec^2 x \, dx = \boxed{\tan x + k}$

82) $\int \tan^3 x \, dx = \int \tan x \,(1 + \sec^2 x) dx = \int \tan x \, dx + \int \tan x \sec^2 x \, dx = \boxed{-\ln|\cos x| + \frac{1}{2}\tan^2 x + k}$

u= tanx
du=sec$^2 x$

83) $\int \tan^7 2x \sec^4 2x \, dx = \int \tan^7 2x \sec^2 2x (1 + \tan^2 2x) \, dx = \int \tan^7 2x \sec^2 2x \, dx + \int \tan^9 2x \sec^2 2x \, dx = \frac{1}{2}\int \tan^7 2x \sec^2 2x \, 2dx + \frac{1}{2}\int \tan^9 2x \sec^2 2x \, 2dx = \boxed{\frac{1}{16}\tan^8 2x + \frac{1}{20}\tan^{10} 2x + k}$

84) $\int \sec x \tan x \, dx = \sec x + k$

85) $\int \sec^3 x \tan x \, dx = \int \sec^2 x \sec x tan \, x \, dx = \boxed{\frac{1}{3}\sec^3 x + k}$

U=sec x
du=sec x tan x

$$\int \boldsymbol{cot^n\,u\,csc^m\,u\,du} \qquad \int \boldsymbol{cot^n\,u\,du} \int \boldsymbol{csc^m\,u\,du}$$

$$\csc^2 u - \cot^2 u = 1$$

86) $\int \csc^2 2x\, dx = \boxed{-\frac{1}{2}\cot 2x + k}$

U=2x
du=2dx

87) $\int \cot 2x \csc^2 2x\, dx = -\frac{1}{2}\int \csc 2x\,(-\csc 2x \cot 2x)dx = \boxed{-\frac{1}{4}\csc^2 2x + k}$

u=csc 2x
du=csc 2xcot 2x 2dx

88) $\int \cot^3 x\, dx = \int \cot x\,(1 + \csc^2 x)dx = \int \cot x\, dx + \int \cot x \csc^2 x\, dx = \boxed{-\ln|\sin x| - \frac{1}{2}\cot^2 x}$

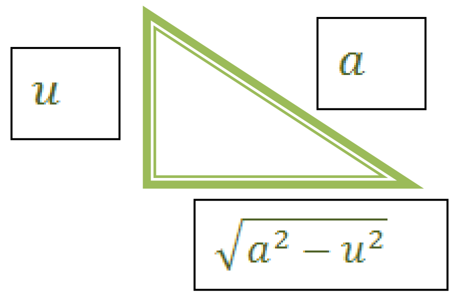
89) $\int \csc 3x \cot 3x\, dx = \boxed{-\csc 3x + k}$

90) $\int \cot^7 5x \csc^2 5x\, dx = -\frac{1}{5}\int \cot^7 5x\,(-\csc^2 5x)5dx = \boxed{-\frac{1}{40}\cot^8 5x + k}$

U=cot 5x
du=$-csc^2 5dx$

Integrales por sustitución trigonométrica

Caso: 1

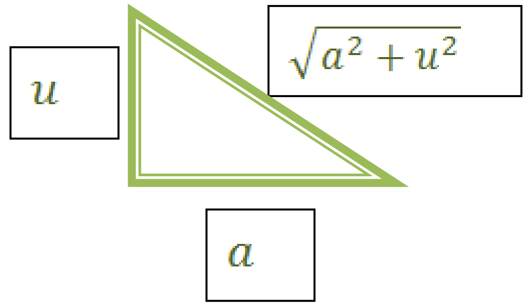


$$\int \frac{xdx}{\sqrt{2^2-x^2}} = \int \frac{2\sin\theta 2\cos\theta}{2\cos\theta} d\theta = 2\int \sin\theta \, d\theta = -2\cos\theta + k \; = \boxed{-\sqrt{2^2-x^2}+k}$$

$$\sin\theta = \frac{u}{a} = \frac{x}{2}$$
$$x =$$
$$dx = 2\cos\theta \, d\theta$$

$$\cos\theta = \frac{\sqrt{2^2-x^2}}{2}$$
$$\sqrt{2^2-x^2} = 2\cos\theta$$

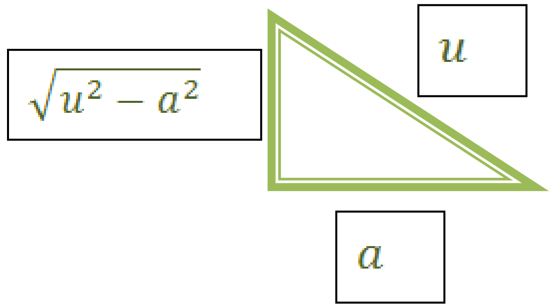
Caso: 2



$$\int x\sqrt{x^2+25}\,dx = \int x\sqrt{x^2+5^2}\,dx = \int 5\tan\theta 5\sec\theta d\theta$$
$$= 25\int \sec\theta\tan\theta\,d\theta = 25\sec\theta + k = \boxed{5\sqrt{x^2+5^2}+k}$$

$$\tan\theta = \frac{x}{5}$$
$$x = 5\tan\theta$$
$$\sec\theta = \frac{\sqrt{x^2+5^2}}{5}$$
$$5\sec\theta = \sqrt{x^2+5^2}$$

Caso: 3



$$\int \frac{x}{\sqrt{x^2 - 2^2}} dx = \int \frac{2\sec\theta \, 2\sec\theta \tan\theta \, d\theta}{2\tan\theta} = 2\int \sec^2\theta \, d\theta$$
$$= 2\tan\theta + k = \boxed{\sqrt{x^2 - 2^2} + k}$$

$\sec\theta = \frac{x}{2}$
$x = 2\sec\theta$
$dx = 2\sec\theta \tan\theta \, d\theta$

$\tan\theta = \frac{\sqrt{x^2 - 2^2}}{2}$
$2\tan\theta = \sqrt{x^2 - 2^2}$

Integrales definidas

91) $\int_1^4 2xdx = 2\int xdx = x^2 = (4^2 - 1^2) = (16-1) = \boxed{15u^2}$

92) $\int_3^4 \frac{2}{x}dx = 2\int_3^4 \frac{1}{x}dx = 2\ln|x| + k = 2(\ln|4| - \ln|3|) = 2\ln\frac{4}{3} + k = 2(0.28) + k = \boxed{0.57u^2}$

93) $\int_1^3 (3x - x^2)\,dx = \int_1^3 3xdx - \int_1^3 x^2 = \frac{3}{2}x^2 - \frac{1}{3}x^3 + k = \frac{3}{2}(3^2 - 1) - \frac{1}{3}(3^3 - 1) = \frac{3}{2}(9-1) - \frac{1}{3}(26-1) = 12u^2 - \frac{25}{3}u^2 = \boxed{3.67u^2}$

94) $\int_0^{\frac{\pi}{2}} \cos x\,dx = \int_0^{\frac{\pi}{2}} sen\ x = \sin\frac{\pi}{2} - \sin 0 = 0.027u^2 - 0 = \boxed{0.027u^2}$

95) $\int e^x - e^{-x}dx = \int e^x\,dx - \int e^{-x}\,dx = e^x + e^{-x} = (e^1 - e^0) + (e^{-1} - e^0) = (2.71-1) + (0.36-1) = 1.70 - 0.63 = \boxed{1.07u^2}$

Integrales por fracciones parciales II

96) $\int \frac{7x+5}{(x+3)(x+1)} dx = \frac{7x+5}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1} = \frac{A(x+1)+B(x+3)}{(x+3)(x+1)}$

$\frac{7x+5}{(x+3)(x+1)} = \frac{A(x+1)+B(x+3)}{(x+3)(x+1)}$

$7x+5 = A(x+1) + B(x+3) = Ax + A + Bx + 3B$
$7x+5 = x(A+B) + (A+3B)$
$7x = x(A+B)$ $\quad 7 = A+B$
$5 = A+3B$
$7 = A+B$
$5 = A+3B$

$A = 8$
$B = -1$

$\int \frac{A}{x+3} dx + \int \frac{B}{x+1} dx = \int \frac{8}{x+3} dx - \int \frac{1}{x+1} dx = \boxed{8\ln|x+1| - \ln|x+1| + k}$

97) $\int \frac{3x-2}{x^3-x^2-2x} dx = \frac{3x-2}{x(x^2-x-2)} = \frac{3x-2}{x(x+1)(x-2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-2}$

$\frac{3x-2}{x(x+1)(x-2)} = \frac{A(x+1)(x-2)+B(x)(x-2)+C(x)(x+1)}{x(x+1)(x-2)}$

$3x-2 = A(x+1)(x-2) + B(x)(x-2) + C(x)(x+1)$
X=0 x+1=0 x-2=0
X=--1 x=+2

Sus x

X=0
$3x-2 = A(x+1)(x-2) + B(x)(x-2) + C(x)(x+1)$
$3(0)-2 = A(0+1)(0-2) + B(0)(0-2) + C(0)(0+1)$
$-2 = -2A$
$A = 1$

Sus x

x=--1

$$3x-2=A(x+1)(x-2)+B(x)(x-2)+C(x)(x+1)$$
$$3(-1)-2=A(-1+1)(-1-2)+B(-1)(-1-2)+C(-1)(-1+1)$$
$$-3-2=B(-1)(-1-2)$$
$$-5=B(1+2)$$
$$-5=3B$$
$$B=-\frac{5}{3}$$

Sus x

X=2

$$3x-2=A(x+1)(x-2)+B(x)(x-2)+C(x)(x+1)$$
$$3(2)-2=A(2+1)(2-2)+B(2)(2-2)+C(2)(2+1)$$
$$6-2=C(2)(2+1)$$
$$4=C(4+2)$$
$$4=6C$$
$$C=\frac{2}{3}$$

$$A=1$$
$$B=-\frac{5}{3}$$
$$C=\frac{2}{3}$$

$$\int \frac{A}{x}dx+\int \frac{B}{x+1}dx+\int \frac{C}{x-2}dx$$
$$=\int \frac{1}{x}dx-\frac{5}{3}\int \frac{1}{x+1}dx+\frac{2}{3}\int \frac{1}{x-2}dx$$
$$=\boxed{\ln|x-|-\frac{5}{3}\ln|x+1|+\frac{2}{3}\ln|x-2|}$$

98) $\int \frac{3}{x^2-1} dx = \int \frac{3}{(x+1)(x-1)} dx = \frac{A}{x+1} + \frac{B}{x-1}$

$$\frac{3}{(x+1)(x-1)} = \frac{A(x-1)+B(x+1)}{(x+1)(x-1)}$$

$$3 = A(x-1)+B(x+1)$$
$$x=1 \qquad x=-1$$

Sus x

X=1

$$3 = A(x-1)+B(x+1)$$
$$3 = A(1-1)+B(1+1)$$
$$3 = 2B$$
$$B = \frac{3}{2}$$

Sus x

X=--1

$$3 = A(x-1)+B(x+1)$$
$$3 = A(-1-1)+B(-1+1)$$
$$3 = -2A$$
$$A = -\frac{3}{2}$$

A=$-\frac{3}{2}$

B=$\frac{3}{2}$

$$\int \frac{A}{x+1} dx + \int \frac{B}{x-1} dx = -\frac{3}{2}\int \frac{1}{x+1} dx - \frac{3}{2}\int \frac{1}{x-1} dx$$
$$= \boxed{-\frac{3}{2}\ln|x+1| - \frac{3}{2}\ln|x-1| + k}$$