INSTITUTO POLITÉCNICO NACIONAL

CECYT No. 1 "GONZALO VÁZQUEZ VELA"

CARTA CESIÓN DE DERECHOS

En la Ciudad de México el día 21 del mes Marzo del año 2012, el (la) que suscribe Oscar Octavio Gutierrez Frías docente (a) del CECyT No.1 Gonzalo Vázquez Vela del Instituto Politécnico Nacional manifiesta que es autor (a) intelectual del presente trabajo presentado en el Evento XIV Congreso Latinoamericano de Control Automático, y cede los derechos del trabajo intitulado Stabilization of the Furuta Pendulum by means of the Direct Lyapunov Method, al Instituto Politécnico Nacional para su difusión, con fines académicos y de investigación.

Los usuarios de la información no deben reproducir el contenido textual, gráficas o datos del trabajo sin el permiso expreso del autor del trabajo. Este puede ser obtenido escribiendo a la siguiente dirección oscargf@sagitario.cic.ipn.mx. Si el permiso se otorga, el usuario deberá dar el agradecimiento correspondiente y citar la fuente del mismo.

Oscar Octavio Gutierrez Frías

Nombre y firma

Stabilization of the Furuta Pendulum by Means of the Direct Lyapunov

Carlos Aguilar-Ibañez *. Miguel S. Suarez-Castañón **
Oscar O. Gutiérrez-Frias ***

* *CIC-IPN, Av. Juan de Dios Bátiz s/n Esq. Manuel Othón de M., Unidad Profesional Adolfo López Mateos, Col. Nueva Industrial Vallejo, Del. Gustavo A. Madero, C.P 07738, México D.F., Phone +(52 55) 57.29.60.00 Ext. 56568 (e-mail: caguilar@cic.ipn.mx)*

** *Escuela Superior de Cómputo - IPN (e-mail: sasuarez@prodigy.net.mx)*
*** *CECyT No. 1 - IPN (e-mail: oscargf@sagitario.cic.ipn.mx)*

Abstract: The Furuta pendulum is stabilized using a nonlinear controller, based on a partial feedback linearization. First only the actuated coordinate of the Furuta pendulum is linearized. Then, the stabilizing feedback controller is obtained by applying the Lyapunov direct method. In other words, using this method we prove local asymptotic stability and demonstrate that the closed-loop system has a large region of attraction. The stability analysis is carried out by means of LaSalle's invariance principle. Numerical simulation to assess the controller effectiveness are included.

Keywords: Furuta Pendulum System; Lyapunov Direct Method; Nonlinear Control.

1. INTRODUCTION

The Furuta pendulum system (FPS) is an under-actuated system and carry out some controlled maneuvers is a rather difficult task. As examples of these maneuver consider stabilizing the FPS around its unstable vertical position, swinging it up from its hanging position to its upright vertical position or, create oscillations around its unstable vertical position (see Fantoni and Lozano (2002); Fradkov et al. (1995); Furuta et al. (1992); Äström and Furuta (2000)). The FPS consists of a horizontal arm, actuated by a direct drive motor, and a free rotating vertical pendulum attached to one end of this arm. Due to the fact that the angular acceleration of the pool cannot be directly controlled, the FPS is an example of an under-actuated mechanical system. This under-actuated system is known to be non-feedback linearizable by means of static state feedback, hence it cannot be linearized by means of dynamic state feedback control, either. Likewise, the system loses controllability when the inverted pendulum crosses the horizontal plane. On the other hand, the linearized model of the FPS is locally controllable around the unstable top equilibrium point (Sira-Ramirez and Agrawal (2004)). Therefore, the control problem can be approximatively solved by a direct pole placement procedure, with the disadvantage of having a very small domain attraction (Barreiro et al. (2002)).

Related to the stabilization of the FPS there are three important issues. The first consists of swinging-up the pendulum, from the hanging position to the upright vertical position. The other issue is stabilizing the FPS with the pendulum at the upright position while the rotating arm rests at the origin, assuming that the initial pendulum deviation angle is restricted to solely move inside the upper-half plane. The third problem deals with making the pendulum to follow stable periodic oscillations. In this work we focus our attention into the second issue mentioned in the previous paragraph. That is, stabilizing asymptotically the **FPS** around its unstable upright position while the rotating arm rest at the origin. The presented controller has two advantages. First, the domain of stability can be estimated and conveniently enlarged up to certain limit, depending on the values of the size and mass of both, the arm and the pendulum. And, because the obtained closed-loop system is locally exponentially stable, the system is robust with respect to small perturbations, like the damping force. Our approach is inspired in the direct Lyapunov method used White et al. (2006,) where a dynamic feedback controller was introduced. However, our control scheme uses only an static feedback controller. As a result, the stability analysis and the domain of attraction estimation were, both, straightforwardly achieved.

The remaining of this work is organized as follows. Section 2 introduces the FPS dynamic model and establishes the control objective. The stability analysis and the construction of the corresponding Lyapunov function are developed in Section 3. Sections 4 and 5 are respectively devoted to present the numerical simulations that assess the effectiveness of the obtained controller and the conclusions.

2. Dynamical Model

The Furuta pendulum, shown in Figure 1, is a mechanical system consisting of an inverted pendulum connected to a horizontal rotating arm acted by a direct current motor. The dynamic non-linear model of the mechanical part of the system, which can be derived from either the Newton or Euler—Lagrange formalisms (Fantoni and Lozano (2002)), is given by:

$$m_1 l_1 L_2 \cos q_1 \ddot{q_2} + \left[J_1 + m_1 {l_1}^2\right]\ddot{q_1}$$
$$-m_1 {l_1}^2 \sin q_1 \cos q_1 {\dot{q}_2}^2 - m_1 l_1 g \sin q_1 = 0;$$
$$\left[J_2 + m_1\left({L_2}^2 + {l_1}^2 \sin^2 q_1\right)\right]\ddot{q_2}$$