THE ROLE OF MATHEMATICAL KNOWLEDGE IN A PRACTICAL ACTIVITY: ENGINEERING PROJECTS AT UNIVERSITY LEVEL

Avenilde Romo Vázquez

Université Paris 7 – Denis Diderot

This paper deals with mathematics education in engineering School. In particular, we are interested in the role these mathematics play in the professional work of engineers. We have chosen to realize this study in Engineers Professional Institute in France. This Institute uses an educational model closely related to the industrial world. During their formation students are required to study a practical question. This activity intends to reproduce the industrial engineer working context. Therefore, our research is focused on these so-called « projects ». In this paper, we present the inquiring methodology and we give an idea of the first outcomes.

The research problem

What place should be given to mathematics in the engineers' formation? Which contents should be approached in this formation and how should they be approached and articulated with other domains of the formation?

These questions have already been asked and treated in different institutions. For example, Belhoste et al. (1994) who studied the formation given by the French *Ecole Polytechnique* between 1794-1994, have shown that the designing of mathematics syllabus recurrently gave rise to discussion.

One of these questions was to decide whether the most important mathematics domain to the engineering formation would be geometry or calculus. This point was clearly related with the most general debate, theory (general method) vs practice (application):

> Pour Lagrange, l'analyse est une méthode générale qui s'applique à la géométrie et à la mécanique, et, sur ce point, il est en accord avec Monge. Mais pour Monge, ce sont les applications qui donnent la vérité de la méthode… » (Belhoste, 1994)

Nowadays, these questions are modified by the technological development, technology taking an increasing place in the engineers' work:

> Before the advent of computers, the working life of an engineer (especially in the early part of his or her career) would be dominated by actually doing structural calculations using pen-and-paper, and a large part of the civil engineering degree was therefore dedicated to giving students an understanding and fluency in a variety of calculational techniques. For the majority of engineers today, all such calculations will be done in practice using computer software. (Kent, 2005)

In other words, the development of powerful software changes the mathematical needs because this software encapsulates some of the usually taught mathematics. Mathematics may even appear to be useless to some engineers.

What is the importance and which role given to the mathematics educations in this new context?

During last years, various researches concerning the nature and the role of the mathematical knowledge in the workplace have been realized (Noss et al., 2000; Kent & Noss, 2002; Magajna & Monagan, 2003; Kent et al. 2004). These researches focused on mathematics and the engineers' work is just a limited amount of those, but from them, we got an outline of the state of the art in this domain about the questions considered as crucial and the theoretical frameworks developed to approach them.

These works point out the existence of gaps between the educational programs and the real world in which the engineers work. For example, the institutional speech asserts that undergraduate engineers need a solid mathematical education, but the researches show that for graduate engineers mathematics is of little use in their professional work.

> Once you've left university you don't use the maths you learnt there, 'squared' or 'cubed' is the most complex thing you do. For the vast majority of the engineers in this firm, an awful lot of the mathematics they were taught, I won't say learnt, doesn't surface again. (Kent and Noss, 2002)

In the same way Prudhomme's research reference recognized the difference between mathematics education in the engineering school (or college) context and mathematics in a workplace context. According to Prudhomme (1999), this gap between educational and working contexts can be explained by the fact that in the first case mathematics follows a disciplinary logic: *The knowledge and its use are built for a disciplinary aim, to answer a prescription of the teacher, without knowing if they really become means to resolve real problems because the solutions have been virtual.* (Prudhomme, 1999). The workplace logic says that knowledge built in the first logic can't be used in the second.

It appears that knowledge built in disciplinary logic is inappropriate for the working context. Noss and Al. (2000) have developed this point in a research focused on mathematics in workplace:

> From a mathematical point of view, efficiency is usually associated with a general method that can then be flexibly applied to a wide variety of problems. This is clearly not the case in the workplace. Even if a number of tasks could potentially be solved with a similar approach, practitioner prefer to use different approaches for each task, partly based on the resources at hand. The crucial point is that orientations such as generalisability and abstraction away from the workplace are not part of the mathematics with which practitioners work.

In our research we intend to contribute to the analysis of the observed gaps and to investigate the role that educational practices and technology play in these gaps. We especially study how one innovative practice in a French engineering Institute intends to articulate theoretical and practical knowledge.

In this paper we mainly focus on an innovative practice, the so-called 'engineering projects' and the methodology that we have implemented to observe it. In the same way we present some data obtained during the observation. The analysis of the obtained data is in process, but we think that the above mentioned analysis can be realized in the frame of the Anthropological Theory of Didactics (ATD) proposed by Chevallard (1999). In this paper we briefly justify our election.

Institutional study

In order to realize our study, we have chosen the Institut Universitaire Professionnalisé d'Evry. This Institute uses an educational model of practical education closely related to the industrial world. This educational model is very interesting for our work, because it is possible to study mathematics taught in an engineering school (college) and at the same time mathematics in use within the practical part of the formation which is closer to the industrial world.

In the first part of our study, we realized an institutional study, for this we have used the Prudhomme's classification, which he introduces in order to analyse the engineering college curricula. In this classification two types of knowledge are considered: purely scientific knowledge and technological knowledge. This last mentioned includes the knowledge related to the technique, in the theoretical or practical sense. For example, Robotics, Mechanics.

Taking into account Martinand's terms reference, Prudhomme organizes these types of knowledge as "disciplines of service" for the purely scientific knowledge and "disciplines of formation" for the technological knowledge. Hence mathematics is taught as a purely scientific domain and yet as a discipline of service, which will become operational in the disciplines of formation.

In our research, we will specially focus in the disciplines of formation, not only in the mathematics as a discipline of service. In particular, we focus on an innovative practice, the so-called *Projects*, which intends to strongly connect the official educational universe of disciplines and the professional world of engineers.

The projects

The projects are realized by a group of three or four students, very independent, respecting a didactical organization which tries to reflect the real organization in workplaces.

The **engineering projects** are carried out by teams of students in their fourth year of engineering school, over five weeks. The subject of every project is open; there is no previous requirement established by client. The final production and the route towards it have to be built together in the same process. Therefore students have to organize and plan their work, to look for solutions; this generally supposes that they adapt or develop their knowledge.

The projects are realized in two phases. After the first one the students write an intermediary report; in this report they describe the pre-project which is in general justified by a study of the subject. They present the technological solution they have chosen among those they have found during their exploratory work. In the second phase the pre-project must lead to a concrete product.
In this kind of projects, the manager is a college teacher, who plays the role of a client who requests a product from a student's group. All the terms and conditions of the project are described in the schedule of conditions (cahier des charges) which is negotiated between the client (teacher) and the distributor (students). The students are supposed to work on their own to come up to the client's request.

The project is assessed from on a double point of view, combining workplace and engineering school requirements. The client must be convinced that the technological solution is the best. But this evaluation is also academic; the students present their work to a jury composed of college teachers. The jury evaluates the use of tools in relation whit knowledge taught in the engineering college. Moreover the students are often asked to justify some of their claims.

Projets Observation methodology

We have realized two observations of the projects. To realize the observation of projects, we used Dumping methodology. In the first phase of project (two weeks) we carried out questionnaires and semi-structured interviews with the students and the clients – tutors. After this phase, we collected institutional data, specifications (document), intermediary reports and documents used for the development of projects. This allowed us to get familiar with projects.

For the second phase we chose only three projects, our aim to be able to realize a deeper and precise observation. To select these projects, we based on the intermediate reports following two criteria: 1) the presence of explicit mathematical knowledge and 2) the project domain such as aeronautics, mechanics, electronics, etc.

In the third week of the project, we met with the students' teams (three teams for three projects) for an interview about the intermediary report; the aim of this interview was to understand the project and to investigate on the role of the identified mathematical contents. We asked the students to do a brief exposition of their project. The aim of this exposition was to identify the role that they were giving to the mathematical content expressed in their intermediary report.

From this, we identified the work division inside the team, and we realized that only one student has the responsibility to develop the mathematical activity. After these meetings, interviews were realized with each student individually.

Some Data from projects observation

In this part, we will present the data collected in the first and second observation in each one of their phases. As well the methodological tools used to obtain the data.

The data from the first and second observation are very different. We consider that this difference comes from the methodology we have employed. The first observation allowed us to assess the methodology; for the second observation the methodological changes allowed us to collect more interesting data.

In the first place we'll present the data obtained in the first phase of the projects, which in our research correspond to the identification of the mathematics used in an explicit way in the first part of the project. We'll do it in a comparative way between the first and second observation; afterwards we show a classification of mathematics identified in this phase.

Later on we'll present the role of the computer software, strongly used in the second phase of the projects. With special attention to what we call 'intermediary element'. The reason of that name is because these kind of elements could be placed in element between the mathematical knowledge taught at College of Engineering and the one used in the practical activity (project).

First Phase

In the first phase of the projects we identify the mathematics used in an explicit way. Here we present the data collected. In order to identify them we have done questionnaires and semi-structured interviews.

Elementary Mathematics (First Observation)

In these projects, mathematics is used in an explicit way, and is relatively elementary such as: functional relations, trigonometry and the application of formulae, calculations. We noticed this use of elementary mathematics in student's answers to questionnaires and in the intermediary reports. Within this observation we did the first interview with 6 teams, the aim (or objective) was to know the possible use of mathematics. It was our first contact with students and projects.

Q1 There will be mathematics in the project?
-mathematics, not, only of **empirical formulae**
- **Calculations** of resistance

Q2 what it is empirical formulae?
-the empirical formulas it is those which one uses in practice
-where it is not necessary make the demonstrations
-You learn how to use them with the experience
- It is not necessary to understand them, you should only know-how,
-and how it is applied

After the interview, we choose 3 teams where the topics were diverse and to observe more than 3 teams was not possible. We did an analysis of intermediary reports within the Anthropological Theory. Other than to relate the techniques described by the students to task out of our field was not easy.

Elementary mathematics and complexes mathematics (Second observation)

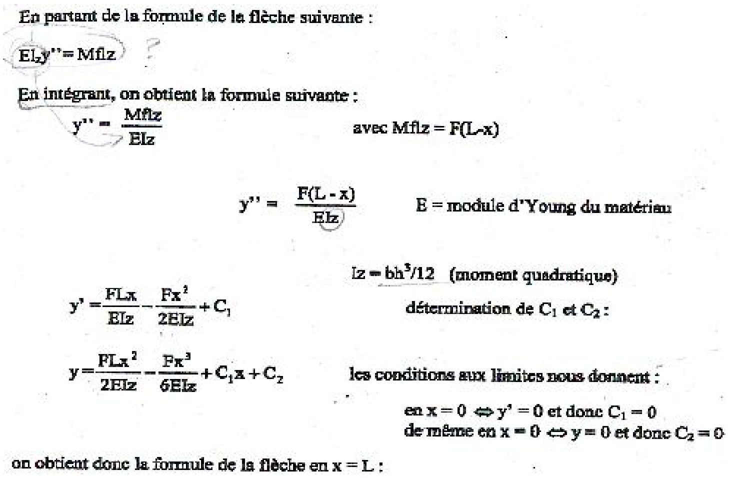
In these projects, elementary mathematics is explicitly used but also some more complex mathematics such as: Differential Equations, transform of Fourier, elements of dimensional analysis, finite elements. In this observation we firstly met 16 student's groups and we studied 16 intermediate reports. The students' answers showed the complexes mathematics used in several projects.

Type of knowledge, tools and skills	Acquired experience in the class
Software used	**Solidworks** –CAO **Ansys for structure calculations**
Calculations made	Fluids Mechanics, vibrations, finite elements
Use of formulae, graphics representation, geometrical representation	Formulae, graphics, abacs, schemes
Other mathematics (functions, linear algebra, differential equations, probability, statistics…)	Functions, Differential Equations
New knowledge for the next step	Yes, documents provided by the tutor concerning new topics
Useful university knowledge for the project	Mechanical Conception, RDM, vibration, Fluids Mechanics
Also useful mathematics lessons for the project	No, Probability don't seem useful at the moment

A questionnaire was done in the first contact with the students. It was the same for the 16 teams and the questions were much more precise. After this questionnaire, we realized some semi-structured interviews based on questionnaire with some of the teams.

In this observation, the intermediate report analysis allowed us to choose three teams of the second phase. We choose three projects of the same domain or field that was important to be able to understand better the projects and the role of the mathematics in these.

In the same way we identified explicit mathematics in the intermediary reports. For example, this extract is from the project: Design and realization of a system of measurement for the blower. The differential equations are used in order to calculate the flexion of blade. This method of calculations is theorized in structural calculus, which is a discipline of formation.


En partant de la formule de la flèche suivante :

$EI_z y'' = Mflz$

En intégrant, on obtient la formule suivante :

$y'' = \frac{Mflz}{EIz}$ avec $Mflz = F(L-x)$

$y'' = \frac{F(L-x)}{EIz}$ E = module d'Young du matériau

$Iz = bh^3/12$ (moment quadratique)

$y' = \frac{FLx}{EIz} - \frac{Fx^2}{2EIz} + C_1$ détermination de C_1 et C_2 :

$y = \frac{FLx^2}{2EIz} - \frac{Fx^3}{6EIz} + C_1 x + C_2$ les conditions aux limites nous donnent :

en $x = 0 \Leftrightarrow y' = 0$ et donc $C_1 = 0$

de même en $x = 0 \Leftrightarrow y = 0$ et donc $C_2 = 0$

on obtient donc la formule de la flèche en $x = L$:

Where does mathematics used in the projects come from?

In both observations, we observed that in certain subjects belonging to the sciences of the engineering field such as the Resistance of Materials, Mechanics or Electronics, there is a strong implication of mathematical elements which is not recognized as such by the students, because they use software that works for them.

After the first observation we classified the different types of mathematics knowledge used in projects.

Mathematical knowledge out of context it is the whole of school mathematical knowledge, i.e. knowledge which is given during the courses of mathematics.

Mathematical knowledge in the context of the engineering sciences is mathematical knowledge present in the constitution of engineering sciences, such as mechanics, dynamics, the structural analysis, the resistance of materials, etc.

The ‘meta-tool’ (Bissell, 2004) belongs to this kind of mathematical knowledge. This term refers to highly sophisticated tools from the mathematical models, used in the fields of electronics, telecommunications and control engineering.

Mathematical knowledge in the context of the engineering practice is that which is used in a systematic way. They are presented in the form of method, like a process recognized socially to solve specific tasks.

Mathematical knowledge in the technological context is that which the use of software mobilizes i.e. mathematical knowledge necessary to uses software.

In fact, we noticed that computer software like RDM (the Resistance of Materials), Solidworks, Catia and others, work on the basis of advanced and complex mathematics. These computers software play a fundamental role in the project development, they allow users to realize calculations, simulations and mechanical systems designs.

In second observation the computers software such as: Matlab, Ansys, Labview are used.

Uses of MatLab

We identified the use of MatLab in the project: Development of a conveyor belt for the aerodynamic study of a light ultra vehicle. To study the aerodynamics' phenomena of a vehicle, it is necessary to reproduce the real conditions. In this project the aim was to build a conveyor belt to reproduce the velocity floor. The group of students designed a conveyor belt as figure 3. The students use Matlab to simulate the system or one of their parts, in particular the simulink option. This option allows to manipulate a system (figure 4).

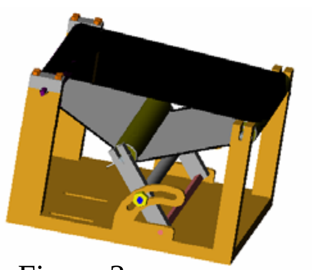


Figure 3

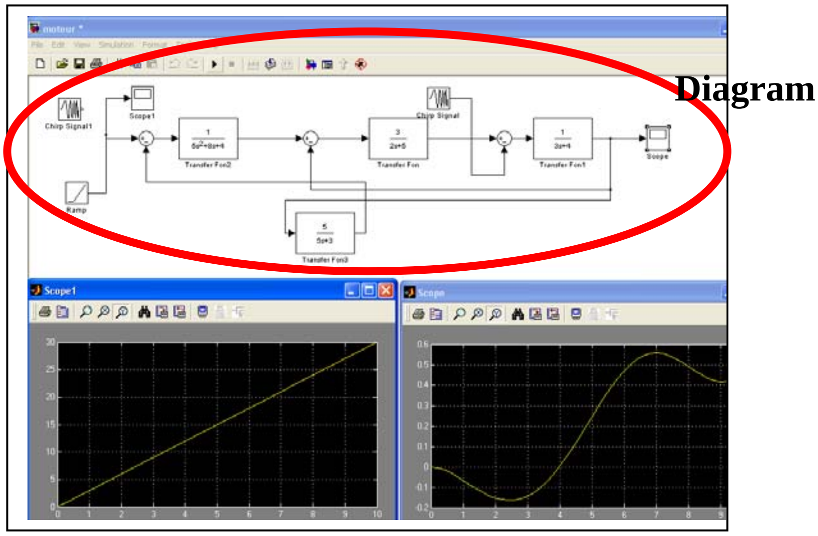


Figure 4

The diagram that appears inside figure 4 is a mathematical model. In this case the diagram models the electric motor of the system. This diagram comes from a discipline of formation: Course d'asservissements. This diagram could be an intermediary element between the mathematical knowledge taught at College of Engineering and the one used in the practical activity (project).

At this time, our work is in the process of analyzing the obtained data. To realize this data analysis according to an institutional study we have decided to use the Anthropological Theory of Didactics (ATD) proposed by Chevallard (1999). For this theory, the knowledge is considered like an emergent of institutionally located practices. The ATD proposes the praxeologies or disciplinary organisation as a tool that allows to model knowledge.

This approach allows us to treat our questions in terms of relations between institutions and to characterize the relations that each one of these institutions have with mathematics, across the notion of institutional relation. This notion allows us to describe and compare the way in which mathematics live in the different devices of formation with the help of the mathematical and didactical praxeologies notions.

Conclusion

The project's observations allow us to study a practical activity closer to professional world within scholar world.

The data obtained show the complexity of the engineer's practical activity. The projects are realized with high technical knowledge and those are a confluence of knowledge and tools. With regard to mathematical knowledge, we noticed that there are several kind of mathematical knowledge:

1) mathematical knowledge out of context

2) mathematical knowledge in context of the engineering sciences, included 'meta-tools'

3) mathematical knowledge in context of the engineering practice

4) mathematical knowledge in technological context

In regard to our previous diagnosis, we need a deeper analysis to assume that the role of the different kinds of mathematical knowledge in the projects is technological in Chevallard's sense, and the mathematical knowledge is the theoretical support of the projects. We stressed that we don't considered the mathematical knowledge in the classical sense.

Bibliography

Bissell, C.C. (2004) Mathematical 'meta-tools' in 20th century information engineering. *Hevelius, 2*, 11-21.

Bissell, C. C. (2003) Histoires, héritages, et herméneutique : la vie quotidienne des mathématiques de l'ingénieur, Annales des Ponts et Chaussées, *107-8*, 4-9

Belhoste, A. (1994) Un modèle à l'épreuve. L'Ecole Polytechnique de 1794 au Second Empire. In Belhoste, B., Dalmedico, A., and Picon, A. (Eds.), *La formation Polytechnicienne* (pp. 9-30). Paris: Dunond

Chevallard, Y. (1999) La recherche en didactique et la formation des professeurs : problématiques, concepts, problèmes. Actes de la X Ecole d'été de Didactique, 98-112. Académie de Caen, France.

Kent, P. (2007) Learning Advanced Mathematics: The case of Engineering courses contribution to the NCTM Handbook chapter: Mathematics thinking and learning at post-secondary level. In Lester, K., F. (Ed.), *Second handbook* *of research on mathematics teaching and learning: a project of the National Council of Teachers of Mathematics*. (pp. 1042-1051). Charlotte, NC: Information Age Pub

Kent, P., & Noss, R. (2001) Investigating the mathematical components of engineering expertise. In Marja van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International group for the Psychology of Mathematics Education Conference*, *1,* 402 Utrecht, The Netherlands.

Kent, P., & Noss, R. (2002) The mathematical components of engineering expertise : The relationship between doing and understanding mathematics. *Proceedings of the IEE Second Annual Symposium on Engineering Education: Professional Engineering Scenarios 2* (pp. 39/1 -39/7)

Noss, R., Hoyles, C., & Pozzi, S. (2000). Working Knowledge: Mathematics in use, In A. Bessot & J. Ridgway (Eds.), *Education for Mathematics in the workplace*, (pp.17-35). Dordrecht: Kluwer Academic Publishers.

Magajna, Z., & Monaghan, J. (2003). Advanced Mathematical Thinking in a Technological Workplace. *Educational Studies in Mathematics*, *52*(2), 101-112.

Prudhomme, G. (1999). Le processus de conception de systèmes mécaniques et son enseignement, Thèse de doctorat pas publiée, Université Joseph Fourier, France