An Efficient Method for Narrowband FIR Filter Design

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Article received on October 16, 1998; accepted on February 22, 1999

Abstract

A new efficient method for implementing narrowband FIR filters is presented. The method is based on an Interpolated filter (IFIR) structure and a cascade of comb and integrator (CIC) structures. The novelty of this technique is that the CIC structure is used as an image suppressor in an IFIR structure. The method is useful for narrowband FIR filter design and some design examples for lowpass and highpass filters are given.

Keywords:

FIR filter, IFIR filters, CIC filters, Narrowband filters.

1 Introduction

Finite impulse response (FIR) filters are often preferred to Infinite impulse response (IIR) filters. FIR digital filters are known to have some very desirable properties such as linear phase, stability, and absence of limit circle; however their application generally requires more computation. A number of techniques have been proposed to reduce the complexity of FIR filters in the past few years. A common approach is to separate the filter transfer function into two or more components having much lower order than the prototype filter (Rabiner and Crochier, 1975; Crochier and Rabiner, 1976; Adams and Willson, 1984; Neuvo *et al.*, 1984; Yong and Yong, 1998).

An alternative method is to manipulate the filter impulse response to reduce its complexity (Bartolo *et al*, 1998).

This paper presents one method based on the first approach.

We consider narrowband filters. As it is known one of the most difficult problems in digital filtering is the implementation of narrowband filters (Rabiner and Crochier, 1975). The difficulty lies in the fact that such narrowband filters require high-order designs to meet the desired frequency response specifications. These high order designs require a large amount of computation and are difficult to implement because of roundoff noise and coefficient sensitivity problems. In this paper we propose an implementation for narrowband digital filters based on Interpolated FIR (IFIR) filters and a cascade of comb and integrator (CIC) structures.

2 IFIR Structure

Lowpass Filters

We consider the design of lowpass narrowband linear phase FIR filter H(z) with cutoff frequencies considerably lower than the sampling rate. An efficient technique for the design and implementation is called the Interpolation FIR (IFIR) technique. IFIR filters were introduced in 1984 (Nuevo *et al.*, 1984) for the design of lowpass filters. The basic idea is to implement a FIR filter as a cascade of two FIR sections, where one section generates a sparse set of impulse response values and the other section performs the interpolation, as presented in Fig.1.a.

The filter G(z) named as the *shaping filter* or *model filter*, is the linear-phase lowpass filter with the passband and stopband cut-off frequencies,

$$\omega_{p,G} = L\omega_p$$

$$\omega_{r,G} = L\omega_r, \qquad (1)$$

where ω_p and ω_s are passbands and stopband frequencies of the prototype filter H(z). The filter $G(z^L)$ is a function of z^L and can be implemented replacing each delay element z^{-1} by an element z^{-L} . This is equivalent to introducing L-1 zeros between each sample of the unit sample response of the filter G(z). L is called the *interpolation factor*. In going from G(z) to $G(z^L)$ the corresponding frequency response is compressed L times. In this way the frequency response in the baseband of $G(z^L)$ is L times narrower, (desired spectrum) and L-1 image frequency responses are produced, (unwanted spectrum), Fig.1.b. In order to suppress the unwanted spectrum a new filter I(z) is cascaded with the filter $G(z^L)$. I(z) is called the *interpolator or image suppressor*, because it is designed to attenuate extra-unwanted passbands of $G(z^L)$.

Therefore the passband and stopband cutoff frequencies of the filter I(z) can be chosen as, (Fig. 1.c):

$$\omega_{p,I} = \omega_p$$

$$\omega_{s,I} = \frac{2\pi}{L} - \omega_s$$
(2)



Fig.1. IFIR filter

The required passband ripple of the prototype filter H(z) must be distributed among the pass-band ripple of the filters G(z) and I(z). To meet the desired specifications we can take the peak passband ripples of G(z) and I(z) to be $R_p/2$, so that the peak passband ripple of the overall system is not greater than R_p . If the stopband attenuation of the filters G(z) and I(z) is A_s , then the overall system has stopband attenuation no larger than A_s , (Vaidyanathan, 1993).

The filter G(z) and I(z) has a much lower order than the prototype filter H(z).

The compression of the spectrum reduces the computational complexity by a factor of L but we must also add in the complexity given by the design of the filter I(z).

Generally it has been shown that IFIR filters require approximately 1/L-th of the adders and multipliers and, in addition, have 1/L-th of the output roundoff noise level and

 $1/\sqrt{L}$ -th of the coefficient sensitivity of an equivalent conventional FIR filter, (Neuvo *et al.*, 1984).

Highpass Filters

The design of a highpass filter can be achieved using the same procedure for the design of a lowpass IFIR filter. The cutoff frequencies for the highpass filter specification ω_p and ω_s are transformed into a corresponding lowpass specification, as follows:

$$\omega_{p} = \pi - \omega_{p}$$
$$\omega_{s} = \pi - \omega_{s} \tag{3}$$

Given the specification (3), a lowpass IFIR filter is designed. For an even interpolation factor L, the unwanted spectrum is at low frequencies and the desired spectrum is at high frequencies. So in order to eliminate the unwanted spectrum it is only necessary to derive a highpass interpolation filter from the lowpass interpolation filter. This is done by changing the sign of every second coefficient, (Fliege, 1998):

$$i_{HP}(n) = (-1)^n i_{LP}(n)$$
 (4)

For an odd interpolation factor L it is necessary to transform both lowpass filters G(z) and I(z) into highpass filters.

3 CIC Structure

Hogenauer, (Hogenauer, 1981), proposed the CIC filter for multirate applications. The system function of the CIC filter is given as

$$H(z) = \left(\frac{1}{L}\frac{1-z^{-L}}{1-z^{-1}}\right)^{K} = \left(\frac{1}{L}\sum_{n=0}^{L-1}z^{-n}\right)^{K}$$
(5)

K is called the *stage*. As is seen in equation (5) all coefficients are equal to 1 and therefore it is not necessary to apply any multiplication.

The frequency response of the CIC filter can be expressed as:

$$H\left(e^{j\omega}\right) = \left\{\frac{\sin\frac{\omega L}{2}}{L\sin\frac{\omega}{2}}e^{-j\omega\left[(L-1)/2\right]}\right\}^{K}$$
(6)

Therefore this is a linear phase lowpass filter. The frequency response has nulls at integer multiples of $2\pi / L$. This makes it a natural candidate to eliminate images introduced by $G(z^L)$, if the baseband of the filter $G(z^L)$ is narrowband. The magnitude responses for two different values of K are given in Fig. 2.



Fig.2. Magnitude responses of CIC filters

4 IFIR-CIC Structure

We propose the use of the CIC structure as an interpolator in the IFIR structure and we call it the IFIR-CIC structure.

The IFIR-CIC structure is shown in Fig.3, and has the advantages of both the IFIR and the CIC structures:

- The order of the shaping filter G(z) is much lower than the order of the prototype filter H(z).
- Both the shaping filter and interpolator are linear phase filters.
- No multipliers are required for interpolator, and are only necessary for the shaping filter.
- No storage is required for interpolator coefficients.
- The structure of CIC filters consists of two basic building blocks: comb and integrators.

The main disadvantages are:

- An increase in the group delay introduced by a two-stage structure.
- The frequency response of CIC filter is fully determined by only L and K resulting in a limited range of filter characteristics so that this structure may be useful only for narrowband prototype filter design.

The proposed algorithm for the design of lowpass filters is given in the following steps:

- 1. Choose L according to the lowpass filter specification.
- 2. Design the lowpass filter G(z).
- 3. Insert L-1 zeros between each value of the impulse response of the filter G(z).
- 4. Use the same value L and choose the value K to design the CIC filter. The starting value for K must be more than the ratio of the order of the interpolation filter and the factor L.

- 5. Cascade the shaping filter and the CIC filter.
- 6. If the magnitude specification is not satisfied try with another value for *K*.

The steps for a design of a highpass filter for the case in which L is even are the same as those shown for a lowpass filter design but for the specification given in (3). Finally the CIC filter is transformed into a highpass filter using (4).

For the case of highpass filter design where L is odd, the same procedure for even L is used but finally both lowpass filters G(z) and I(z) are transformed into highpass filters.

5 Examples of the Filter Design

Here we present different examples of the filter design and compare them with the ordinary IFIR structures.

The Remez algorithm is used for the filter G(z) design. The MATLAB program is implemented for the design of corresponding lowpass and highpass filters using the IFIR-CIC structure.

Example 1.

We design a lowpass filter with the passband cutoff frequency

 $\omega_p = .01$, and the stopband frequency $\omega_s = .02$. Passband ripple is $R_p=.25$ and the stopband attenuation is -60 dB. The design of the H(z) using the Remez algorithm gives the order of the filter N=490. Magnitude response of the filter is given in Fig.4. An IFIR structure with the interpolation factor L=5 gives the order of the shaping filter 110 and the order of the interpolator 15. An IFIR-CIC structure uses the same value of L and the value K=5. The results of the design for both filters are given in Fig.5. Passband and the stopband zoom are given in Fig. 6.



Fig.3. IFIR-CIC structure



Fig.4. The prototype filter H(z)

Fig. 5. IFIR and IFIR-CIC filters, (L=5, K=5)



Fig.6. Passband and stopband zoom, (L=5, K=5)

If the interpolation factor L=10 is used in the IFIR structure, the order of the shaping filter is 55 and the order of the interpolation filter is 31. The corresponding IFIR-CIC structure uses the same value L and the value K=3. The design results for both filters are given in Fig.7 and Fig.8.



Fig. 7. IFIR and IFIR-CIC filters, (L=10, K=3)



Fig.8. Passband and stopband zoom, (L=10, K=3)

Example 2.

We design a highpass filter with passband cutoff frequency $\omega_p = .99$, and stopband cutoff frequency $\omega_s = .97$. Passband ripple is $R_p = .25$ and the stopband attenuation is -60 dB. The design of the filter with the Remez algorithm gives the order of the filter N=248. Magnitude response is given in Fig.9. Using L=10, the shaping filter has order 30 and the interpolator has order 31. The same value of L and the value K=5 are used in the IFIR-CIC structure. Resulting filter designs are given in Fig.10. Passband and the stopband zoom are given in Fig. 11.



Fig.9. The prototype Filter



Fig. 10. IFIR and IFIR-CIC filter, (L=10, K=5)



Fig.11. Passband and stopband zoom, (L=10, K=5)

6 Conclusion

This paper presents a new technique for narrowband lowpass and highpass filter design. The method is based on the application of an IFIR structure where the interpolation filter is a CIC filter. We call this structure IFIR-CIC structure. The structure has the same advantages as an IFIR structure: the component filters have a much lower order than the prototype filter. The design results in less computation complexity, which means a decrease in the number of adders and multipliers, of roundoff noise level and of coefficient sensitivity.

The IFIR-CIC structure also has the main advantages of a CIC structure, for example no multipliers and no storage are required for interpolator coefficients. The CIC filters consist of two basic building blocks: comb and integrators. The overall result is less computation complexity for an IFIR-CIC structure as compared to the IFIR structure. The main disadvantage is that the frequency response of the CIC filter is fully determined by only L and K resulting in a limited range of filter characteristics. This implies that the proposed structure is useful only for narrowband prototype filter design. The presented examples of the design show that the IFIR-CIC structure also gives a better stopband attenuation in than the corresponding IFIR structure. In the case when the passband of the overall filter is deteriorated it is necessary to apply a compensator, (Chu, Burrus, 1984). In the case when the order of the shaping filter is still high it is necessary to apply a multistage structure, (Vaidyanathan, 1993). The proposed structure can also be of use in the design of minimum phase filters, (Dolecek and Diaz, 1998).

Acknowledgement

This work was supported by a CONACYT grant, number 211290-5-3009PA.

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