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Oscar Octavio Gutiérrez-Frias

Nombre y firma

Lyapunov Method for the Controlling of the Two Wheels Inverted pendulum

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***Abstract*— In this paper, a nonlinear controller for the stabilization of the two wheels inverted pendulum is presented. Firstly, by suitable partial feedback linearization that allows to linearize only the actuated coordinate system, we proceed to find a function Lyapunov in conjuntion with LaSalle's invariance principle. Based on this candidate function, we derive a stabilizing controller in such away that the closed-loop system is locally asymptotically stable around its unstable equilibrium point, with a computable domain attraction.**

Keywords: Under Actuated System, Two Wheels Inverted Pendulum, Lyapunov Method, Non-Linear Control.

I. Introduction

In last decade, the family the inverted pendulums attracted the attention of control researchers as a benchmark for evaluating a wide range of non-linear control methods [1], [2], [3], [4], [5]. The Two Wheels Inverted Pendulum (**TWIP**) is another challenging category of the inverted pendulum system, as seen by growing list of articles, for examples: [6], [7], [8], [9], [10], [11], [12], to mention only a few references. The device consists of a free vertical rotating pendulum with two wheels actuated by DC motors, the torque of the motors is used with input control. Besides, this nonlinear system permits us to illustrate a simplified version of the well known *Segway* [13]. The main control objective is to drive the pendulum to the rest upright position with the wheels to the rest origin. Since the angular acceleration of vertical pendulum cannot be directly controlled it is a example of an underactuated mechanical system. Exist many works related with the problem control of this system but a detailed review of the state of the art of the problem here treated is beyond the scope of this work. However, we refer the interested reader to the following references: [14], [15], [16], [17], [18], [19], [20].

In this article, we develop a control law for the Two Wheels Inverted Pendulum, which allows to stabilize locally and asymptotically the closed-loop system, with a computable domain attraction. The control strategy is based on the Lyapunov method, in combination with LaSalle' invariance theorem.

The rest of the paper is organized as follows. Section II presents the model of the **TWIP** and a partial feedback linearization is introduced. Section III is devoted to building a candidate Lyapunov function to obtain a nonlinear controller for the stabilization of the **TWIP**. The analysis stability of the closed-loop system is also presented in the same section. Section IV presents some computer simulations, whereas in Section V is devoted to presenting the conclusions. Finally, an Appendix is presented.

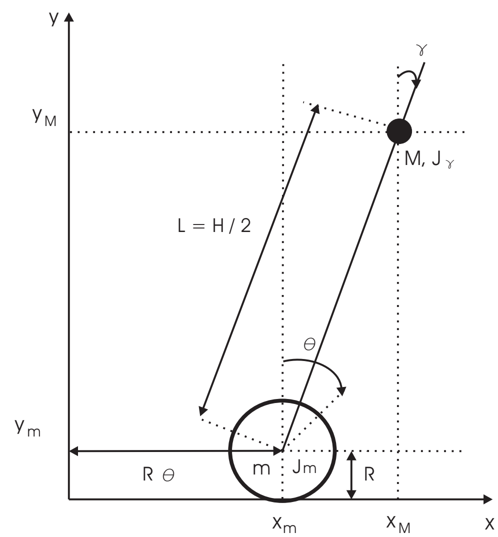

Fig. 1: Two Wheels Inverted Pendulum system.

II. Dynamical Model

Consider the Two Wheels Inverted Pendulum system[1] as shows in Figure 1.

The nonlinear model can be obtained from the Euler-Lagrange equations in [21]. It is given by

$$\begin{array}{l} [(2m+M)R^2+2J_m]\ddot{\theta}+MLRcos\gamma\ddot{\gamma}-MLR\gamma^2 sin\gamma=f \\ MLRcos\gamma\ddot{\theta}+[ML^2+J_\gamma]\ddot{\gamma}-MgLsin\gamma=0 \end{array} \tag{1}$$

where θ is the rotacional angle of wheels, γ is the rotacional angle of the pendulum, and f is the voltage applied to the motors, acting as the control input. Parameters M y J_γ stand for the mass and the inertia of the pendulum; m, J_m y R are mass, inertia moment

[1] The pendulum is restricted to move in the xy-plane.